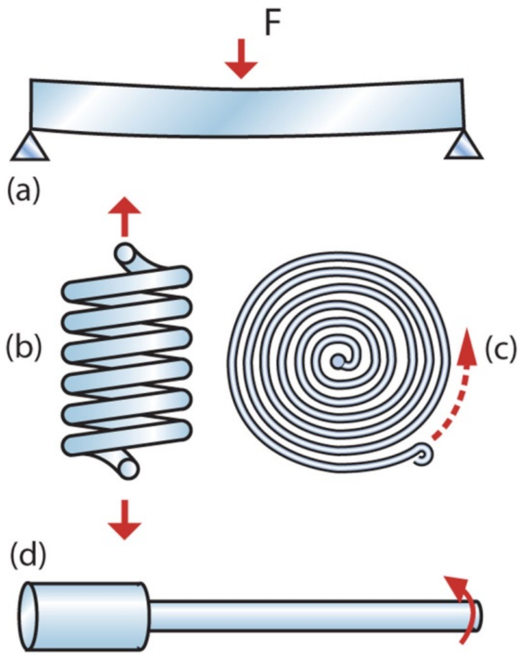
CES EduPack Case Studies:

Mechanical Applications



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GRANTA
TEACHING RESOURCES

This case study document is part of a set based on Mike Ashby's books to help introduce students to materials, processes and rational selection.
The Teaching Resources website aims to support teaching of materials-related courses in Design, Engineering and Science.
Resources come in various formats and are aimed primarily at undergraduate education.

About These Case Studies

These case studies were created with the help of Prof. Yves Brechet, Prof. David Embury, Dr. Norman Fleck, Dr. Jeff Wood, and Dr. Paul Weaver. Thanks also to Mr. Ken Wallace, the Director of the Cambridge University Engineering Design Centre and to the Engineering and Physical Sciences Research Council for their support of research into Materials Selection.

We are indebted to Ericka Jacobs for her help with proof reading the final manuscript, editing the graphics, and laying-out the entire book.

Contents

1 Introduction

This document is a collection of case studies in Materials Selection. They illustrate the use of a selection methodology, and its software-implementation, the CES EduPack™. It is used to select candidate materials for a wide range of applications: mechanical, thermal, electrical, and combinations of these. Each case study addresses the question: out of all the materials available to the engineer, how can a short list of promising candidates be identified?

The analysis, throughout, is kept as simple as possible whilst still retaining the key physical aspects which identify the selection criteria. These criteria are then applied to materials selection charts created by CES EduPack, either singly, or in sequence, to isolate the subset of materials best suited for the application. Do not be put off by the simplifications in the analyses; the best choice of material is determined by function, objectives and constraints and is largely independent of the finer details of the design. Many of the case studies are generic: those for beams, springs, flywheels, pivots, flexible couplings, pressure vessels and precision instruments are examples. The criteria they yield are basic to the proper selection of a material for these applications.

There is no pretense that the case studies presented here are complete or exhaustive. They should be seen as an initial statement of a problem: how can you select the small subset of most promising candidates, from the vast menu of available materials? They are designed to illustrate the method, which can be adapted and extended as the user desires. Remember: design is open ended — there are many solutions. Each can be used as the starting point for a more detailed examination: it identifies the objectives and constraints associated with a given functional component; it gives the simplest level of modeling and analysis; and it illustrates how this can be used to make a selection. Any real design, of course, involves many more considerations. The 'Postscript' and 'Further Reading' sections of each case study give signposts for further information.

1.1 The Design Process

1. What are the steps in developing an original design?

   Answer

   - Identify market need, express as *design requirements*
   - Develop *concepts*: ideas for the ways in which the requirements might be met
   - *Embodiment*: a preliminary development of a concept to verify feasibility and show layout
   - *Detail design*: the layout is translated into detailed drawings (usually as computer files), stresses are analyzed and the design is optimized
   - *Prototyping*: a prototype is manufactured and tested to confirm viability

1.2 From Design Requirements to Constraints

2. Describe and illustrate the “Translation” step of the material selection strategy.

   Answer

   Translation is the conversion of design requirements for a component into a statement of function, constraints, objectives and free variables.

FUNCTION	What does the component do?
OBJECTIVE	What is to be maximized or minimized?
CONSTRAINTS	What non-negotiable conditions must be met?
FREE VARIABLE	What parameters of the problem is the designer free to change?

2 Material for Oars

Credit for inventing the rowed boat seems to belong to the Egyptians. Boats with oars appear in carved relief on monuments built in Egypt between 3300 and 3000 BC. Boats, before steam power, could be propelled by poling, by sail and by oar. Oars gave more control than the other two, and their military potential was well understood by the Romans, the Vikings and the Venetians.

Records of rowing races on the Thames in London extend back to 1716. Originally the competitors were watermen, rowing the ferries which carried people and goods across the river. Gradually gentleman became involved (notably the young gentlemen of Oxford and Cambridge), sophisticating both the rules and the equipment. The real stimulus for development of high performance boats and oars came in 1900 with the establishment of rowing as an Olympic sport. Since then both have exploited to the full the craftsmanship and materials of their day. Consider, as an example, the oar (Figure 2-1).

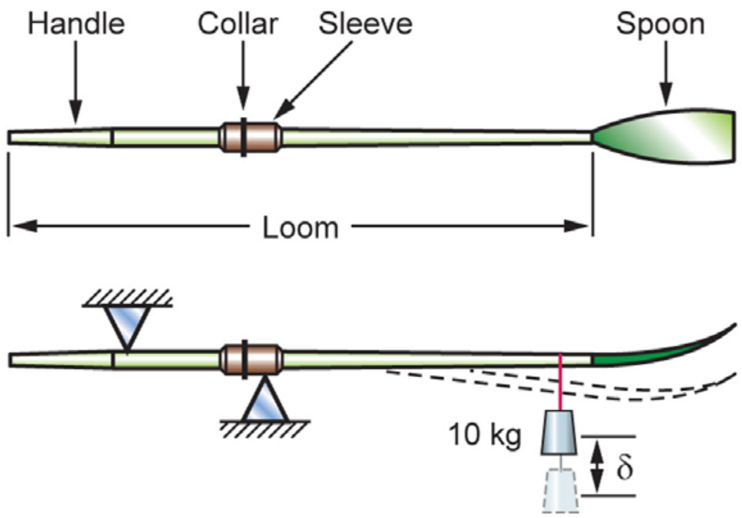

Figure 2-1 Oars are designed on stiffness, measured in the way shown in the lower figure; and they must be light

The requirements of the design are summarized in Table 2-1.

Table 2-1 The design requirements

FUNCTION	Light, stiff beam
OBJECTIVE	Minimize mass
CONSTRAINTS	(a) Length L specified (b) Bending stiffness S specified (c) Toughness, G_c > 1 kJ/m2 (d) Cost, Cm < 100 USD/kg

2.1 The Model

Mechanically speaking, an oar is a beam, loaded in bending. It must be strong enough to carry the bending moment exerted by the oarsman without breaking, it must have just the right stiffness to match the rower's own characteristics and give the right 'feel', and — very important — it must be as light as possible. Meeting the strength constraint is easy. Oars are designed on stiffness, that is, to give a specified elastic deflection under a given load. The upper part of Figure 2-1 shows an oar: a blade or 'spoon' is bonded to a shaft or 'loom' which carries a sleeve and collar to give positive location in the rowlock. The lower part of the figure shows how the oar stiffness is measured: a 10 kg weight is hung on the oar 2.05 m from the collar and the deflection at this point is measured. A soft oar will deflect nearly 50 mm; a hard one only 30 mm. A rower, ordering an oar, specifies how hard it should be.

The oar must also be light; extra weight increases the wetted area of the hull and the drag that goes with it. So there we have it: an oar is a beam of specified stiffness and minimum weight.

The mass, m, of the oar — treated as a solid cylinder — is

$$m = AL\rho = \pi R x^2 L\rho \qquad \text{(M 2.1)}$$

where A is the area of the cross-section and ρ the density of the material of which it is made. This is the objective function — the quantity to be minimized. The stiffness of the beam is:

$$S = \frac{CEI}{L^3} \qquad \text{(M 2.2)}$$

Where E is the modulus, $I$is the second moment of area of the beam and C is a constant (roughly 24 — the material selection is independent of its value). For a solid cylinder

$$I = \frac{\pi R^4}{4} \qquad \text{(M 2.3)}$$

The length $L$and stiffness S are specified: they are constraints. The free variable is the radius R. We use equations (M 2.2) and (M 2.3) to eliminate Rin equation (M 2.1) giving

$$m = 2\left(\frac{SL^5}{\pi C}\right)^{1/2}\left(\frac{\rho}{E^{1/2}}\right) \qquad \text{(M 2.4)}$$

The mass m of the oar is minimized by choosing materials with large values of

$$\boxed{M_1 = \frac{E^{1/2}}{\rho}} \qquad \text{(M 2.5)}$$

The design requirements list two further constraints — on toughness G_c and cost C_m. These are frequently taken for granted — the designer subconsciously rejects materials which are too brittle (glass, for instance) or too costly (platinum). It is better to make them explicit. We therefore require that the limits set out in Table 2-1 on page 4 are met.

2.2 The Selection

The selection has two stages. Figure 2-2 shows the first, plotted at Level 2. It is a chart of Young's modulus, E, against density, ρ. A selection line for the index is shown on it. It identifies three classes of material: woods, carbon- and glass-fiber reinforced polymers and certain ceramics (Table 2-2). Ceramics meet the first set of design requirements, but are brittle; a ceramic oar, if dropped, might shatter.

Shock-resistance requires adequate toughness, G_c, (not just fracture toughness, KIC). A useful rule-of-thumb for this is to choose materials with a toughness, G_c, such that

$$G_c = \frac{K_{IC}^2}{E} \geq 1\ kj/m^2 \qquad \text{(M 2.6)}$$

We require a second stage as shown in Figure 2-3: a chart of toughness against cost. The toughness axis is created by generating a user-defined property combination with KIC and E, as per equation (M 2.6). A box selection specifies materials with

$G_c > 1kJ/m^2$ and $C_m < 100\ USD/kg$

Ceramics are eliminated because they are brittle. The recommendation is clear. Make your oars out of wood or — better — out of CFRP.

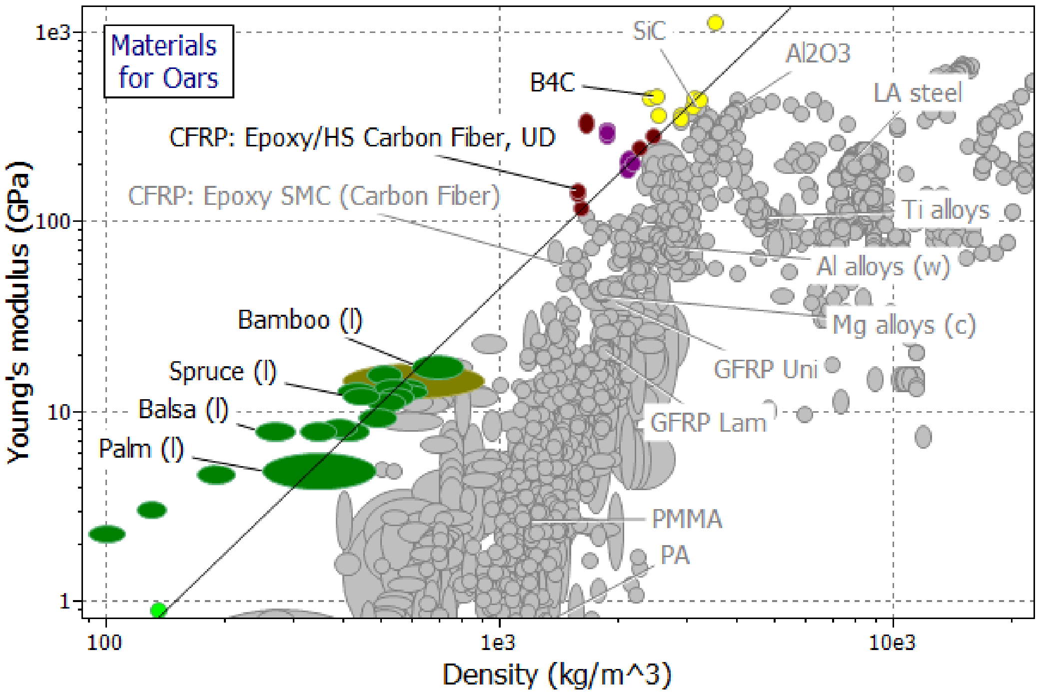

Figure 2-2. A chart of modulus, E, against density, ρ. The selection line is placed at M_1 = 0.006 GPa$^{1/2}$/(kg/m^3).

Table 2-2. Materials for oars

MATERIAL	M_1 (GPa)$^{1/2}$/(kg/m^3)	COMMENT
Woods	0.005 – 0.008	Cheap, but not easily controlled and low G_c
CFRP	0.004 – 0.008	As good as wood, more control of properties
GFRP	0.002 – 0.004	Cheaper than CFRP but lower G_c
Ceramics	0.004 – 0.008	Good M but brittle — eliminated by low G_c

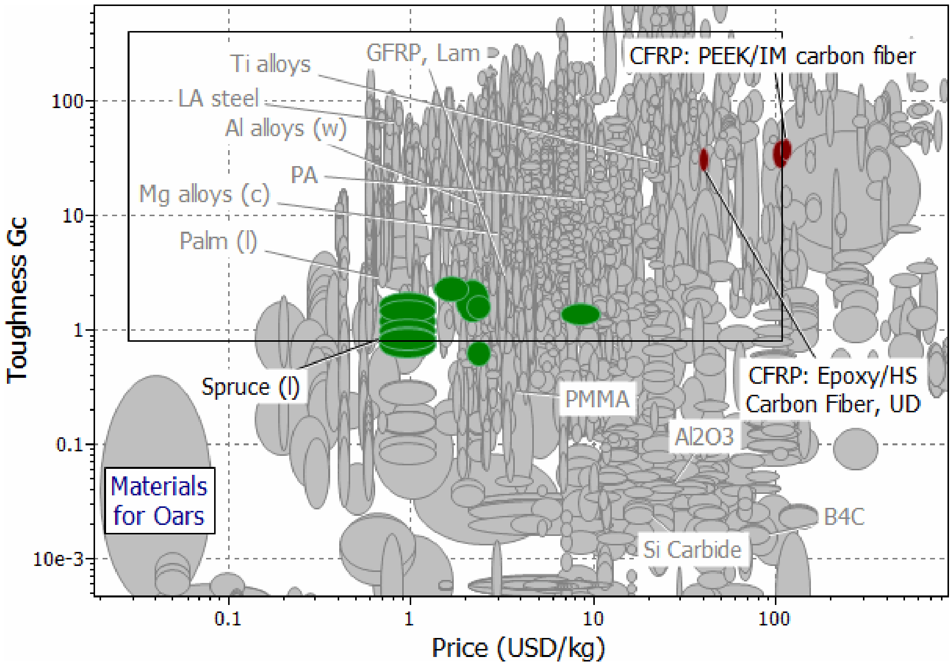

Figure 2-3. An example chart of toughness, G_c, against cost/kg, C_m, made at Level 3 of the database. The box isolates materials with G_c > 1 kJ/m^2 and C_m < 100 USD/kg (75 GBP/kg).

2.3 Postscript

Now we know what oars *should* be made of. What, in reality, is used? Racing oars and sculls are made of wood or of a high performance composite: carbon-fiber reinforced epoxy, CFRP.

Wooden oars are made today, as they were 100 years ago, by craftsmen working largely by hand. The shaft and blade are of Sitka spruce from the northern US or Canada, the further north the better because the short growing season gives a finer grain. The wood is cut into strips, four of which are laminated together to average the stiffness. A strip of hardwood is bonded to the compression-side of the shaft to add stiffness, and the blade is glued to the shaft. The rough oar is then shelved for some weeks to settle down, and then finished by hand cutting and polishing. The final spruce oar weighs between 4 and 4.3 kg, and costs (in 1994) about £150 ($250).

Composite blades are a little lighter than wood, for the same stiffness. The component parts are fabricated from a mixture of carbon and glass fibers in an epoxy matrix, assembled and glued. The advantage of composites lies partly in the saving of weight (typical weight: 3.9 kg) and partly in the greater control of performance: the shaft is molded to give the stiffness specified by the purchaser. At a price, of course: a CFRP oar costs about £300 ($450).

Could we do better? The Chart shows that wood and CFRP offer the lightest oars, at least when normal construction methods are used. Novel composites, not at present shown on the chart, might permit further weight saving; and functional-grading (a thin, very stiff outer shell with a low density core) might do it. But both appear, at present, unlikely.

2.4 Further Reading

Redgrave, S, '*Complete Book of Rowing*', (1992), Partridge Press, London.

3 Materials for Buildings

The most expensive thing that most people buy is the house they live in. Roughly half the cost of a house is that of the materials of which it is made, and they are used in very large quantities (family house: around 200 tonnes; large apartment block: around 20,000 tonnes). The materials are used in three ways: structurally, to hold the building up; as cladding, to keep the weather out; and as 'internals', to insulate against heat, sound, and so forth.

Consider the selection of materials for the structure of a building (Figure 3-1). They must be stiff, strong, and cheap. Stiff, so that the building does not flex too much under wind loads or internal loading. Strong, so that there is no risk of collapse. And cheap, because such a lot of material is used. The structural frame of a building is rarely exposed to the environment, and it is not generally visible. So criteria of corrosion resistance or appearance, are not so important. The design goal is simple: strength and stiffness at minimum cost.

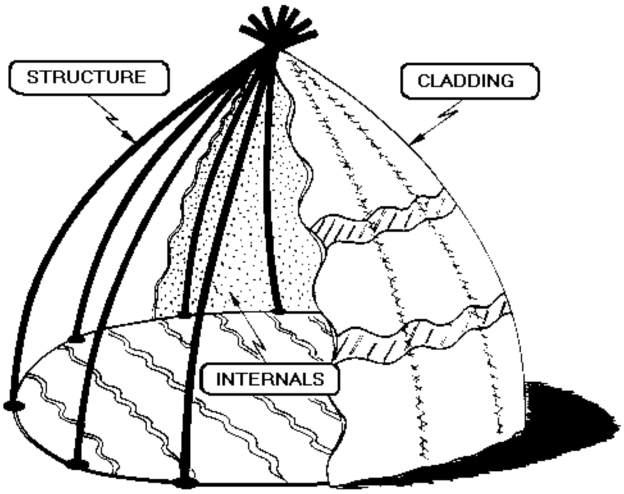

Figure 3-1. The materials of a building perform three broad functions. The frame gives mechanical support, the cladding excludes the environment, and the internal surfacing controls heat, light, and sound.

Table 3-1. The design requirements

FUNCTION	Structural material to carry bending moments
OBJECTIVE	Minimize cost per unit of function
CONSTRAINTS	Adequately stiff Adequately strong

3.1 The Model

The performance indices for cheap, stiff, and strong materials are standard ones — they can be found in the tables of indices through the *Help*-button. The critical components in a building are loaded either in bending (floor joists, for example) or as columns (the vertical members). The two indices that we wish to maximize are:

$$\boxed{M_1 = \frac{E^{1/2}}{\rho C_m}} \qquad \textit{(M 3.1)}$$

and

$$\boxed{M_2 = \frac{\sigma_f^{2/3}}{\rho C_m}} \qquad \textit{(M 3.2)}$$

where E is Young's modulus, ρ is the density, C_m the cost/kg of the material and σ_f is the failure strength, which we shall take to be the compressive strength, σ_c .

3.2 The Selection

Figure 3-2 and Figure 3-3 show the appropriate charts plotted at Level 3. The first shows the modulus E plotted against cost per unit volume C_m ρ. The selection line has the appropriate slope of 2. It isolates concrete, stone, brick, softwoods, cast irons, and the cheaper steels. The second chart shows compressive strength plotted against cost. The selection line, M_2 this time, gives almost the same selection. The values of the performance indices are listed in Table 3-2. No surprises. They are precisely the materials of which buildings are made.

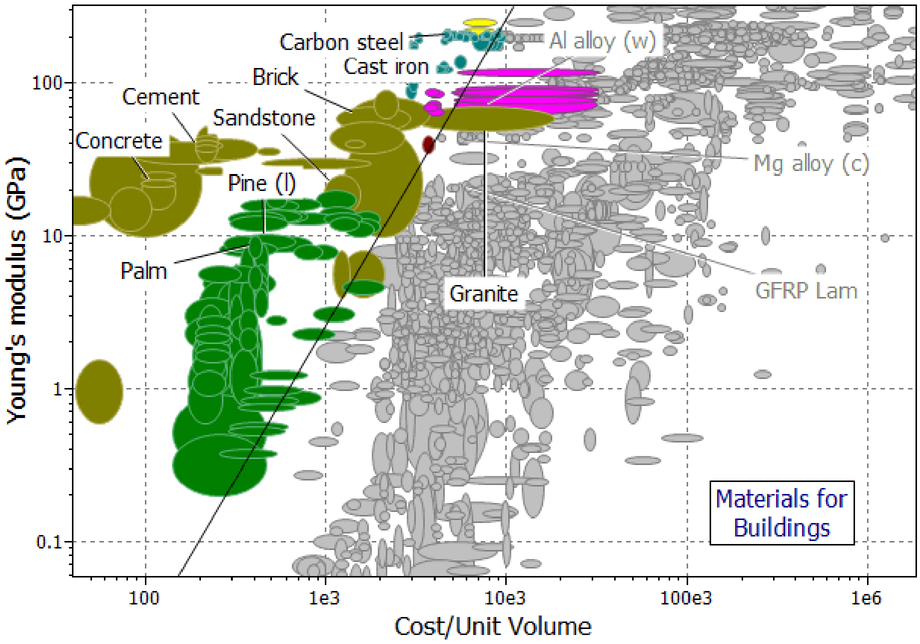

Figure 3-2. The modulus, E, plotted against cost per unit volume, $C_m\rho$, using 'All bulk materials' record subset

Table 3-2. Structural materials for buildings with typical values

MATERIAL	M_1 $(GPa^{1/2}/(USD/m^3))$	M_2 $(MPa^{2/3}/(USD/m^3))$	COMMENT
Concrete	0.04	0.08	Use in compression only
Brick	0.02	0.045	
Stone	0.015	0.055	
Woods	0.015	0.08	Tension and compression, with freedom of section shape
Cast Iron	0.005	0.02	
Steel	0.003	0.021	
Reinforced Concrete	0.02	0.06	

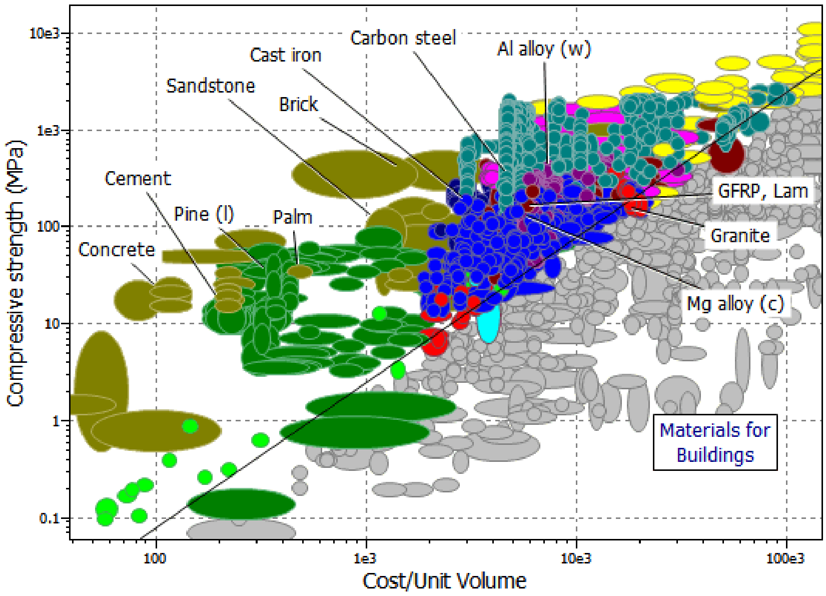

Figure 3-3. The compressive strength, σc , plotted against cost per unit volume, $C_m\rho$, using the 'All bulk materials' record subset

3.3 Postscript

It is sometimes suggested that architects live in the past; that in the late 20th Century they should be building with fiberglass (GFRP), aluminum alloys and stainless steel. Occasionally they do, but Figure 3-2 and Figure 3-3 give an idea of the penalty involved. To achieve the same stiffness and strength will cost between 5 and 10 times more than the conventional materials. Civil construction is materials-intensive: the cost of the material dominates the cost of bridges, roads and buildings, and the quantity used is enormous. Then only the cheapest of materials qualify, and the design must be adapted to use them. Concrete, stone and brick have strength only in compression; the form of the building must use them in this way (columns, arches). Wood, steel and reinforced concrete have strength both in tension and compression, and steel, additionally, can be given efficient shapes (I-sections, box sections, tubes); the form of the building made from these has much greater freedom.

3.4 Further Reading

Cowan, HJ and Smith, PR (1988) '*The Science and Technology of Building Materials*', Van Nostrang-Reinhold, NY

4 Materials for Springs

Springs come in many shapes and have many purposes. One thinks of axial springs (a rubber band, for example), leaf springs, helical springs, spiral springs, torsion bars and so on (Figure 4-1). Regardless of their shape or use, the best material for a spring is that which can store the greatest elastic potential energy per unit mass (or volume), without failing. The performance indices derived below, can be used to identify materials which satisfy the design specification summarized in Table 4-1.

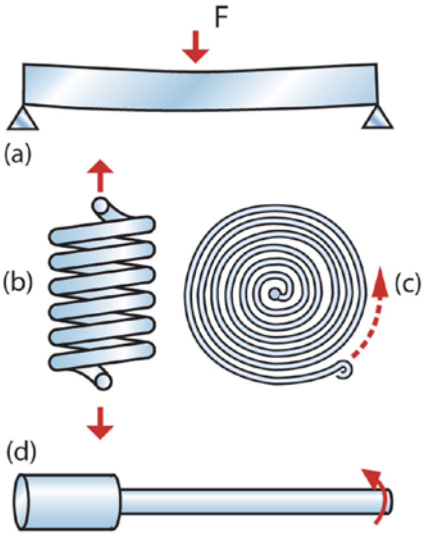

Figure 4-1. Springs have many shapes, but all perform the same function: that of storing elastic energy.

Table 4-1. The design requirements

FUNCTION	Elastic Spring
OBJECTIVE	(a) Maximum stored elastic energy/unit volume (b) Maximum stored elastic energy/unit mass
CONSTRAINTS	No failure by yield, fatigue or fracture (whichever is more restrictive) Adequate toughness (Gc > 1 kJ/m2) Reasonable cost per unit weight (Cm < 100 GBP/kg)

4.1 The Model

The primary function of a spring is that of storing elastic energy and releasing it again when required. The elastic energy stored per unit volume in a block of material stressed uniformly to a stress σ is:

$$W_v = \frac{1}{2}\frac{\sigma^2}{E} \qquad \textit{(M 4.1)}$$

where E is the Young's modulus. It is this that we wish to maximize. The spring will be damaged if the stress σ exceeds the yield stress or failure stress σ_f. So the constraint is $\sigma \leq \sigma_f$. The maximum energy density is therefore:

$$W_v = \frac{1}{2}\frac{\sigma_f^2}{E} \qquad \textit{(M 4.2)}$$

Torsion bars and leaf springs are less efficient than axial springs because some of the material is not fully loaded: the material at the neutral axis, for instance, is not loaded at all. For solid torsion bars[1]

$$W_v \approx \frac{1}{3}\frac{\sigma_f^2}{E} \qquad \textit{(M 4.3)}$$

and for leaf springs loaded in pure bending the maximum energy storage is

$$W_v = \frac{1}{4}\frac{\sigma_f^2}{E} \qquad \textit{(M 4.4)}$$

But, as these results show, this has no influence on the choice of material. The best material for springs, regardless of the way in which they are loaded, is that with the biggest value of

$$\boxed{M_1 = \frac{\sigma_f^2}{\rho E}} \qquad \textit{(M 4.5)}$$

If mass matters rather than volume, we must divide this by the density ρ (giving energy stored per unit mass), and seek materials with high values of

$$\boxed{M_2 = \frac{\sigma_f^2}{\rho E}} \qquad \textit{(M 4.6)}$$

[1] The value of the constant (1/3, 1/4) in equations (M 4.3) and (M 4.4) can be increased by prestressing: the maximum working stress of torsional and leaf springs is increased by straining the material beyond its elastic limit and then releasing the load.

4.2 The Selection

The selection of materials for springs of minimum volume is shown in Figure 4-2. Here the modulus of rupture, σ_{MOR}, has been used as the measure of the failure strength σ_f. The chart shows σ_{MOR} plotted against modulus, E. A family of lines of slope 1/2 link materials with equal values of $M_1 = \sigma_f^2/E$. Those with the highest values of M_1 lie towards the top left. The heavy line is one of the family. It is positioned at 10 MJ/m^3 such that a small subset of materials is left exposed. They include *high-strength steel* (spring steel, in fact) lying near the top end of the line, and, at the other end, *rubber*. But certain other materials are suggested too: *GFRP* (now used for truck leaf springs), *titanium alloys* (good but expensive), *glass fibers* (used in galvanometers) and — among polymers — *nylon* (children's toys often have nylon springs). The procedure identifies a candidate from almost every material class: metals, glasses, polymers, elastomers and composites. A protective stage, limiting the values of the toughness G_c ($G_c = K_{IC}^2$ / E) and the cost C_m to the those listed in the design requirements, has been added (Figure 9.3). The selection results are shown in Table 4-2.

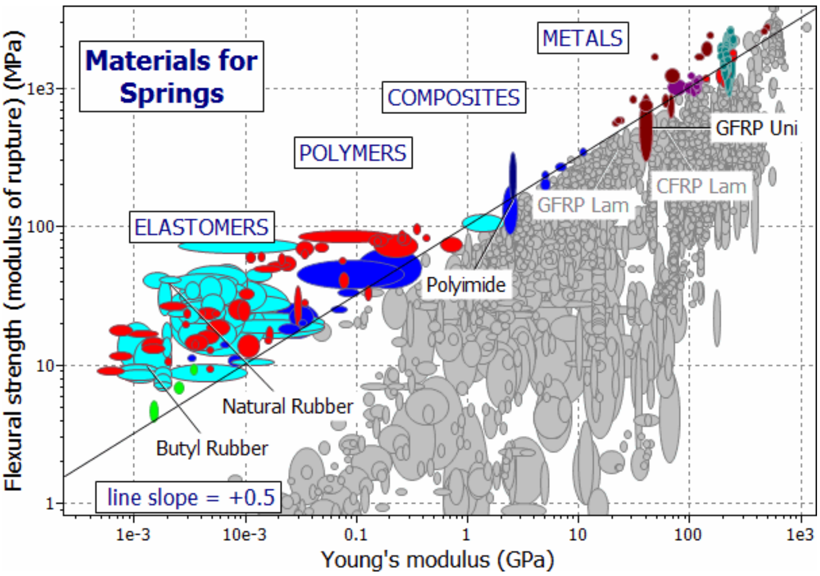

Figure 4-2. A chart of the modulus of rupture, σ_{MOR}, against Young's modulus, E. The diagonal line shows M_1.

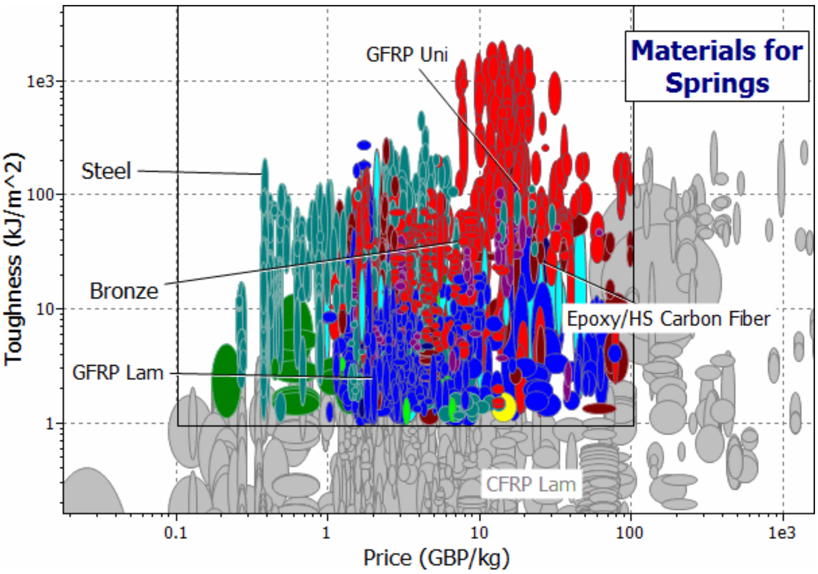

Figure 4-3. A 'protective' chart of the toughness, G_c, against cost per unit weight, C_m. The box restricts the selection to materials with $G_c > 1$ kJ/m^2 and $C_m < 100$ GBP/kg. The currency unit is easily changed in 'Options'.

Table 4-2. Materials for efficient springs of low volume

MATERIAL	$M_1 = \frac{\sigma_f^2}{f} (MJ/m^3)$	COMMENT
Spring Steel	15 – 25	The traditional choice: easily formed and heat treated.
Ti Alloys	15 – 20	Expensive, corrosion-resistant.
CFRP	15 – 20	Comparable in performance with steel; expensive.
GFRP	10 – 12	Almost as good as CFRP and much cheaper.
Glass fibers	30 – 60	Brittle in tension, but excellent if protected against damage; very low loss factor.
Nylon	1.5 – 2.5	The least good; cheap and easily shaped, but high loss factor.
Rubber	20 – 60	Better than spring steel; but high loss factor.

Materials selection for light springs is shown in Figure 4-3. It is a chart of σ_{MOR}/ρ against E/ρ, where ρ is the density. Lines of slope 1/2 now link materials with equal values of

$$M_2 = \left(\frac{\sigma_f}{\rho}\right)^2 / \left(\frac{E}{\rho}\right) = \frac{\sigma_f^2}{E\rho}$$

One is shown at the value M_2 = 2 kJ/kg. The new selection is listed in Table 4-3. Composites, because of their lower densities, are better than metals. Elastomers are better still (you can store almost 8 times more elastic energy per unit weight in a rubber band than in the best spring steel). Elastomeric springs are now widely used in aerospace because of their low weight and high reliability. Wood — the traditional material for archery bows, now appears in the list.

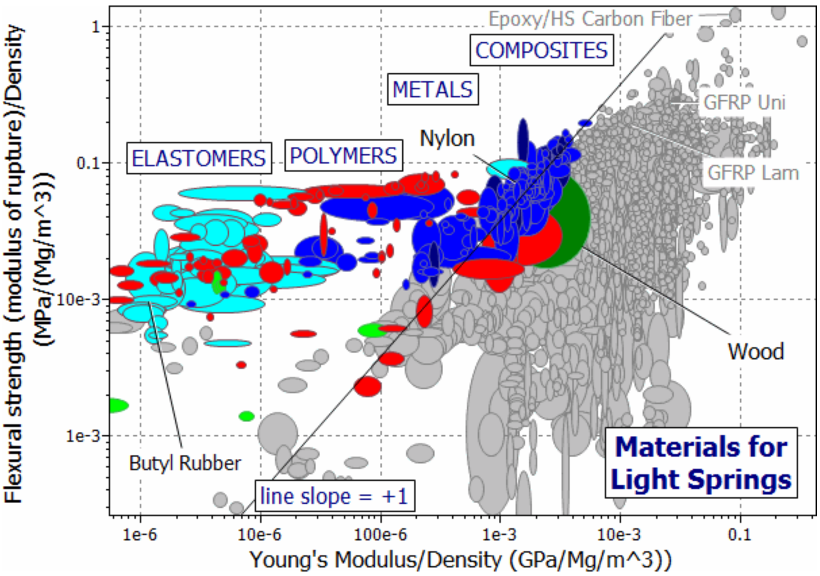

Figure 4-4. A chart of σ_{MOR}/ρ against E/ρ. The diagonal line is a contour of M_2.

4.3 Postscript

Many other considerations enter the choice of a material for a spring. Springs for vehicle suspensions must resist fatigue (the selection should then be made with the endurance limit, σ_e, replacing the modulus of rupture, σ_{MOR}). Valve springs for internal combustion engines must cope with elevated temperatures; here a strength-at-temperature is needed. The mechanical loss coefficient is important in springs which are loaded dynamically: polymers have high loss factors and therefore dissipate energy when they vibrate; metals, if strongly hardened, do not. Polymers, because they creep, are unsuitable for springs which carry a steady load, though they are good for catches and locating-springs which spend most of their time unstressed. Springs made from unprotected carbon-steel fail rapidly in a chemically corrosive environment.

The least expensive solution to this problem is to plate or polymer-coat them to provide a corrosion barrier, but if the coating is damaged, failure can follow. The more expensive solution is to make the spring from an intrinsically corrosion-resistant material: stainless steel, copper alloys, nickel and cobalt alloys, titanium alloys, reinforced polymers, GFRP or CFRP. If high thermal or electrical conduction is required, copper-beryllium alloys are the best choice. But these are secondary selection-criteria. The primary criterion, important in defining the initial subset of materials, remains that defined by equations (M 4.5) and (M 4.6).

Table 4-3. Materials for efficient light springs

MATERIAL	$M_2 = \frac{\sigma_f^2}{E\rho}\,(kJ/kg)$	COMMENT
Spring Steel	2 – 3	Poor, because of high density.
Ti Alloys	2 – 3	Better than steel; corrosion-resistant; expensive.
GFRP	4 – 8	Better than steel; expensive.
CFRP	3 – 5	Better than steel; less expensive than CFRP.
Glass fibers	10 – 30	Brittle in torsion, but excellent if protected.
Woods	1 – 2	On a weight basis, wood makes good springs.
Nylon	1.5 – 2	As good as steel, but with a high loss factor.
Rubber	20 – 50	Outstanding; 10 times better than steel, but with high loss factor.

4.4 Further Reading

Boiten, RG (1963) ‘*The Mechanics of Instrumentation*’, Proc. IMechE. vol 177, No. 10, 269–288.

Hayes, M (1990) ‘*Materials Update 23: Springs*’, "Engineering", May (1990), pp 42–43.

5 Safe Pressure Vessels

Pressure vessels (Figure 5-1), from the simplest aerosol can to the biggest boiler, are designed for safety. Two ways of doing this are to arrange that they either yield or leak before they break. Small pressure vessels are usually designed to allow general yield at a pressure which is too low to propagate any crack the vessel may contain ('yield before break'). The distortion caused by yielding is easy to detect and the pressure can be released safely. With large pressure vessels this may not be possible. Instead, safe design is achieved by ensuring that the smallest crack that can propagate unstably has a length greater than the thickness of the vessel wall ('leak before break'). The leak releases pressure gradually and thus safely. The two criteria lead to slightly different performance indices, but essentially the same choice of materials. What are they?

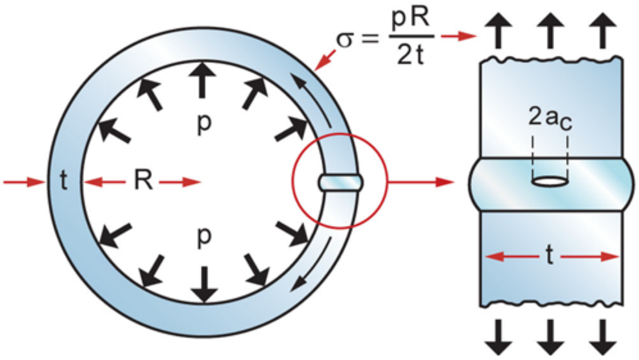

Figure 5-1. A pressure vessel. It must be assumed that flaws pre-exist. The design and choice of material must ensure that they do not propagate.

Table 5-1. The design requirements

FUNCTION	Contain pressure, p
OBJECTIVE	Minimize cost Minimize weight
CONSTRAINTS	(a) Must yield before break, or (b) Must leak before break

5.1 The Model

We idealize the pressure vessel as a thin-walled sphere of radius R and wall thickness t. (The material selection aspects of the problem are independent of shape, so we choose the shape which offers the simplest analysis.) The mass of the vessel is:

$$m = 4\,\pi\,R^2\,t\,\rho \qquad \textit{(M 5.1)}$$

In pressure vessel design, the wall thickness, t, is chosen so that, at the working pressure p, the stress is less than the yield strength, σ_y, of the wall.

$$\sigma = \frac{pR}{2t} \le \sigma_y \qquad \textit{(M 5.2)}$$

A small pressure vessel can be examined ultrasonically, or by X-ray methods, or proof tested, to establish that it contains no crack or flaw of diameter greater than $2a_c$. The stress required to make such a crack propagate is

$$\sigma_f = \frac{C\ K_{IC}}{\sqrt{\pi\ a_c}} \qquad \textit{(M 5.3)}$$

where C is a constant near unity. Safety obviously requires that the working stress is also less than the fracture stress of equation (M 5.3); but greater security is assured by requiring that the crack will not propagate even if, in an overload, the stress reaches the general yield stress. Then the vessel will deform stably in a way which can be detected. This condition is expressed by requiring that σ_f be greater than the yield stress, σ_y, giving

$$\pi a_c \leq C^2 \left[\frac{K_{IC}}{\sigma_y}\right]^2 \qquad \textit{(M 5.4)}$$

The tolerable crack size is maximized by choosing a material with the largest value of

$$\boxed{M_1 = \frac{K_{IC}}{\sigma_y}} \qquad \textit{(M 5.5)}$$

Large pressure vessels cannot always be X-rayed or tested ultrasonically; and proof-testing them may be impractical. Further, cracks can grow slowly because of corrosion or cyclic loading, so that a single examination at the beginning of service life may not be sufficient. Then safety can be assured by arranging that a crack just large enough to penetrate both the inner and the outer surface of the vessel is still stable, because the leak caused by the crack can be detected. This is achieved by setting a_c in equation (M 5.3) equal to t/2:

$$\sigma_f^* = \frac{C\ K_{IC}}{\sqrt{\pi\ t/2}} \qquad \textit{(M 5.6)}$$

The wall thickness t of the pressure vessel was, of course, designed to contain the pressure p without yielding. From equation (M 5.2), this means that

$$t \leq \frac{p\ R}{2\sigma_y} \qquad \textit{(M 5.7)}$$

Substituting this into the previous equation (with $\sigma_f^* = \sigma_y$) gives

$$p = \frac{4\ C^2}{\pi\ R}\left(\frac{K_{IC}^2}{\sigma_y}\right) \qquad \textit{(M 5.8)}$$

The maximum pressure is carried most safely by materials with the large values of

$$\boxed{M_2 = \frac{K_{IC}^2}{\sigma_y}} \qquad \textit{(M 5.9)}$$

Both M_1 and M_2 could be made large by making the yield strength of the wall, σ_y, very small. Lead, for instance, has high values of both M_1 and M_2. But you would not choose lead for a pressure vessel because, to carry a useful pressure, the wall would have to be very thick. To minimize the wall thickness (equation (M 5.7)) we must maximize either M_1 or M_2, *and* the index

$$\boxed{M_3 = \sigma_y} \qquad \textit{(M 5.10)}$$

So far we have focused on stationary pressure vessels, for which weight is unimportant. Those which move (spacecraft, rocket casings, submersibles) must be both safe and light. The mass of the vessel is found by substituting equation (M 5.7) into (M 5.1)

$$m = 2\,\pi\,R^3\,p\left(\frac{\rho}{\sigma_y}\right)$$

Therefore, the mass can be minimized by choosing materials with large values of the index

$$\boxed{M_4 = \frac{\sigma_y}{\rho}} \qquad \textit{(M 5.11)}$$

5.2 The Selection

Safe minimum weight selection is achieved in two stages. For stationary vessels, both of these can be shown on the same chart (Figure 5-2). It shows the fracture toughness, K_{1C}, plotted against elastic limit, σ_y. The two criteria M_1 and M_2 appear as lines of slope 1 and ½ respectively. Take 'leak before break' as an example. The diagonal line corresponding to $M_2 = K_{1C}^2 / \sigma_y = C$ excludes everything but the toughest steels, copper and aluminum alloys, though some polymers nearly make it (pressurized lemonade and beer containers are made of these polymers). The vertical line shows the 'wall thickness' index M_3: efficient materials lie to the right of this line, and above the diagonal M_2 index line. The selection is listed in Table 5-2.

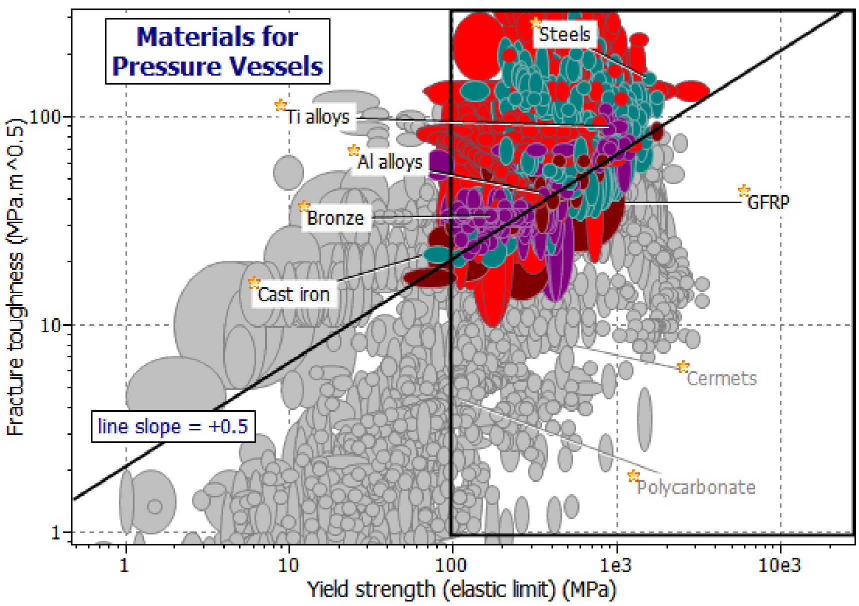

Figure 5-2. A chart of fracture toughness K_{IC} against elastic limit σ_y, using the generic record subset, showing the indices M2, and M3 = 100 MPa.

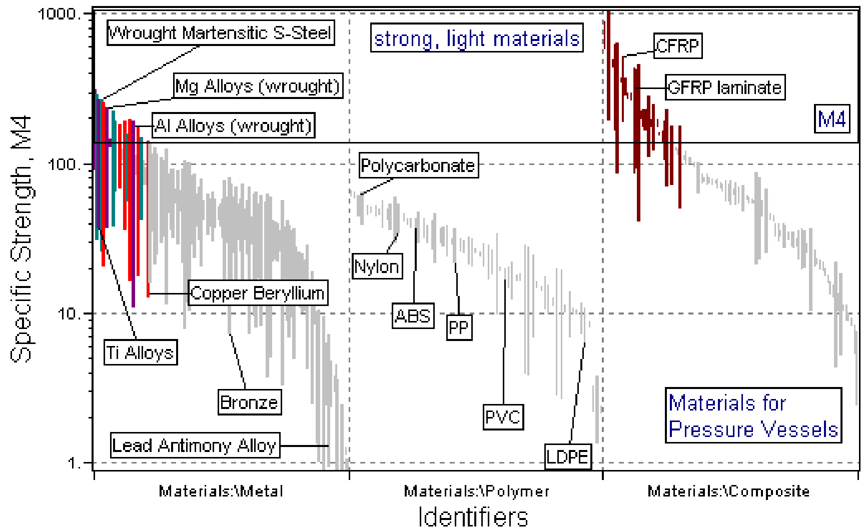

Figure 5-3. A chart showing M4. Strong, light materials lie near the top.

Table 5-2. Materials for safe pressure vessels

MATERIAL	$M_2 = \frac{K_{IC}^2}{\sigma_2}$ **(MPa.m)**	**COMMENT**
Tough steels	10 – 20	These are the pressure-vessel steels, standard in this application.
Tough copper alloys	10 – 20	OFHC hard drawn copper.
Ti-Alloys	8 – 15	Good performance but expensive
High Strength Al-Alloys	5 – 10	Less good than steel, copper or titanium

Light pressure vessels require that M_4, rather than M_3, be maximized. This can be achieved by adding a second stage in which $M_4 = \sigma_y / r$ is plotted as a bar-chart (Figure 5-3). The candidate materials are those with large values of both M_2 and M_4. They are listed in **Table** 5-3.

Table 5-3. Materials for light pressure vessels

MATERIAL	$M_2 = \frac{K_{IC}^2}{\sigma_2}$ **(MPa.m)**	**COMMENT**
CFRP	3 – 5	Considerable weight saving over metals, but expensive
GFRP	3 – 5	Considerable weight saving over metals
Tough steels	10 – 20	These are the pressure-vessel steels, standard in this application.
Mg-alloys	3 – 5	Low toughness makes Mg a poor choice
Ti-alloys	8 – 15	Good performance, but expensive
High strength Al-alloys	5 – 10	Less good than steel, copper or titanium

In practice, large pressure vessels are always made of steel. Those for models — for instance a model steam engine — are copper. Copper is favoured in the small scale application because of its greater resistance to corrosion. When weight is important, copper alloys are not a good choice; aluminum alloys, GFRP and CFRP offer the best combination of toughness, strength and low density.

5.3 Postscript

Boiler failures used to be commonplace — there are even songs about it. Now they are rare, although when safety margins are pared to a minimum (rockets, experimental aircraft design), pressure vessels still occasionally fail. This (relative) success is one of the major contributions of fracture mechanics to engineering practice.

5.4 Further Reading

Background in fracture mechanics and safety criteria can be found in these books:

- Brock, D (1984) '*Elementary Engineering Fracture Mechanics'*, Martinus Nijoff, Boston.
- Hellan, K (1985) '*Introduction to Fracture Mechanics*', McGraw-Hill.
- Hertzberg, RW (1989) '*Deformation and Fracture Mechanics of Engineering Materials'*, Wiley, NY.

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