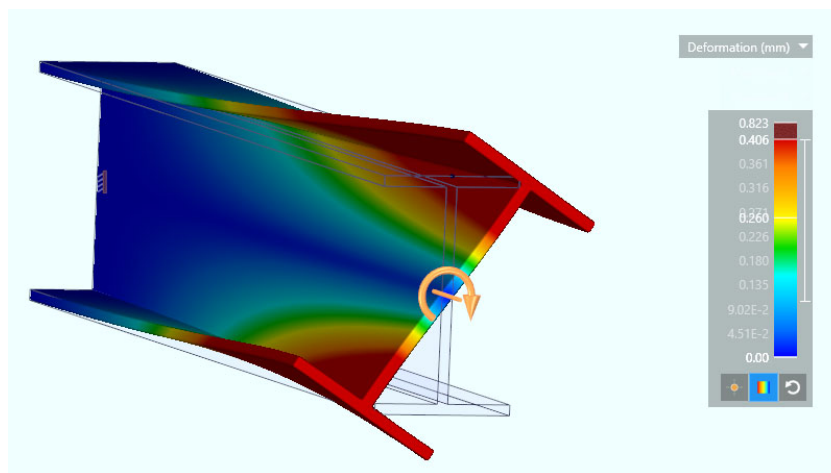

Material Properties and Structural Sections

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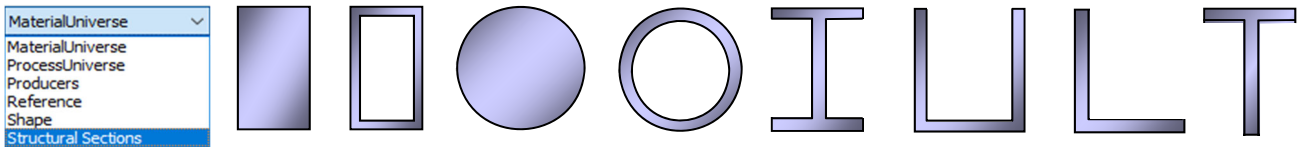
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Summary

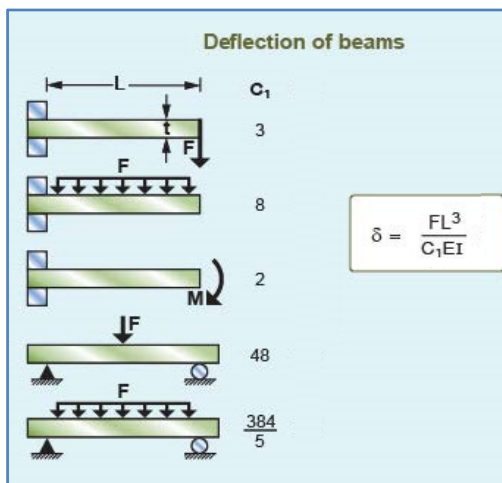
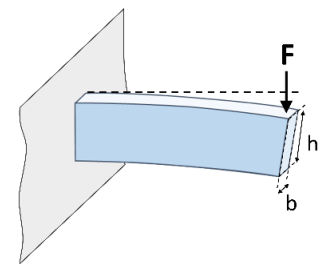
When materials are used for structural support, such as in buildings and general construction, the shape is as fundamental as the material properties themselves. This shape determines how effective the material is fulfilling its structural function, for instance beams supporting loads in bending. The influence of the shape on performance is usually separated from the visual process of material selection in CES EduPack. However, there exists a data-table of standard structural sections for a limited number of materials. This Case study session deals with selection of structural sections and complementary Finite Element simulations. A good material choice is beneficial as input in a simulation where details about the shape can be explored and the free design parameter used in CES EduPack can be determined.

1. Scope

Materials used for structural support, such as in buildings and general construction, rely on their shape. It is as important as the material properties themselves. The cross-section determines how effective the material is at fulfilling its structural function (e.g., supporting loads during bending). There are a number of standard sections available, that all have their merits in terms of structural efficiency. This reflects how well they perform carrying loads compared to, say, a solid square beam as a reference. I, U and L-sections are good in bending but poor in torsion while hollow circular and box sections carry both torsion and bending well.

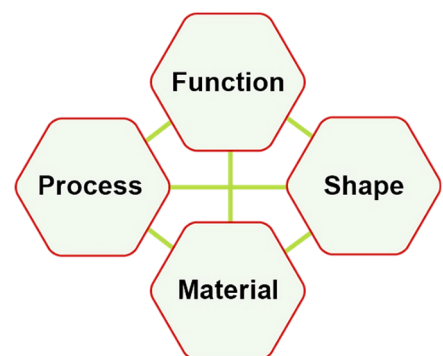


Structural sections are a design case where architecture meets engineering for the built environment. Their application is connected to safety in buildings as well as minimizing material use and ultimately cost. Two of the most important situations for structural sections in buildings are compressive load and bending in various forms. One simple example is a cantilever, which is a *fixed-end beam loaded in bending* by vertical forces, as shown to the right. Strength is important, of course, but stiffness is the key performance-limiting property. The less deformation in a building (e.g., floor joists), the better it is.



The deflection of a cantilever depends on the applied load and a combination of material properties and shape, mainly cross-section profile. It is worth mentioning that the deflection displays the same principal dependence in terms of shape (represented by the second moment of area, I) and material (represented by Young's modulus, E) for many common bending cases. Note, in the summary to the left, that only the value of the constant (C_1) varies for point loads or distributed loads (and combinations thereof) on the beam. This constant will not affect the selection outcome using material property charts. We also note that a column in compression behaves similar to a beam in bending, for a stiffness-limited design, since it is prone to buckling at critical deflections (Euler) [1]. A simple cantilever example is thus relevant and illustrative for many cases.

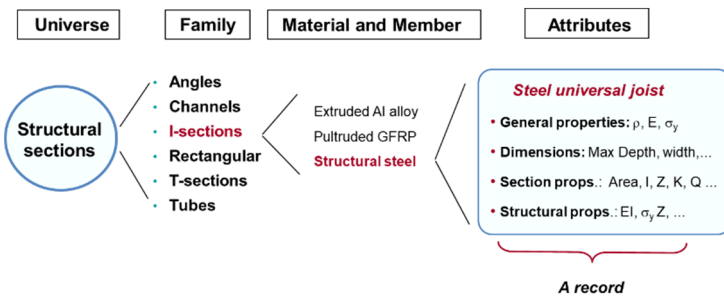
Assuming that a main function can be identified and separated in the material selection (e.g. beam in bending), a chart of pure material properties can normally be used to rank the options with respect to their performance objective. This requires that the cross section shape is fixed (e.g. a square) and that a free design parameter, such as area, has been eliminated from the performance index, to be determined later. A low-modulus material will need a larger area and a stiff material can be made thinner to deliver the same performance. The material selection is then made visually based on the merits of material properties alone for the specific objectives (e.g. light and stiff).



In this case study, we elaborate how shape can be dealt with explicitly during material selection for structural sections and explore how the data-table of structural sections may be used for this purpose. We also look at how Finite Element simulations can complement material selection in a design process.

2. What can EduPack do?

There are nearly 2000 standard structural sections for selected materials available in a separate data-table in all *Advanced* (Level 3) databases and in the specialized *Architecture* database at Level 2. The existing set of materials consists of structural steel, extruded aluminum alloy, pultruded GFRP and wood. They have the shapes shown in the previous page at the top – rectangular, tube, I-sections, U-sections (Channels), L-shapes (Angles) and T-sections. The data organization is in the same “Tree” style as the MaterialUniverse and ProcessUniverse. The first level of the tree – the families – refers to shape. The second level is the material – we include only four for which standard sections are widely available.



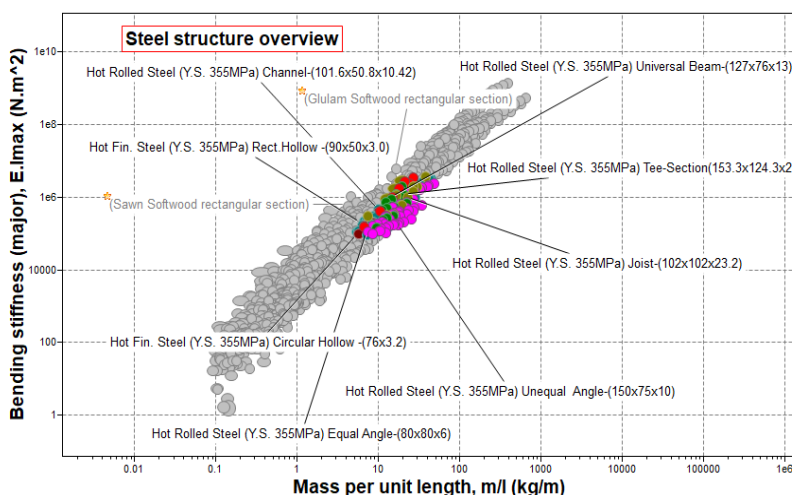
The structural sections come in many different sizes, and for each size there is a set of attributes. The records in this database contain: material properties, dimensions, section properties such as the second moment of area I and section modulus Z , as well as structural properties (e.g., as flexural rigidity, $E \cdot I$ and failure moment, $\sigma_y \cdot Z$).

Some attributes in the structural sections data-table reflect material properties, such as *Price* (\$/kg) or *Yield strength*, σ_y (MPa), which can be plotted in charts in the usual way. However, most properties are specific to the structural section of the datasheet. To visualize structural section data in a chart, we must therefore remember that the shape is not separated from the material properties in the usual way. An overview of only *steel structures* (Rectangular, Tube, I-Section, Channel, Angle and T-Section) is shown, below. The bending stiffness (major) is plotted. For these structures, mass per unit length is more relevant than density. Non-steels are greyed out by a Tree stage filter.

Hot Rolled Steel (Y.S. 355 MPa) Universal Beam (610x305x149)

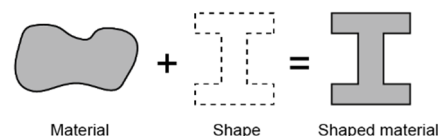
General			
Embodied energy, primary production	① 23	- 31	MJ/kg
Price	① 0.404	- 0.448	USD/kg
Recycle fraction	① 0.9	- 0.95	
Safety factor	① 1	- 1.05	
Dimensions			
Maximum Depth, D	① 0.607	- 0.615	m
Maximum Width, B	① 0.301	- 0.309	m
Section			
Section area, A	① 0.0169	- 0.0214	m ²
Second moment of Area (major), I _{max}	① 0.00111	- 0.00141	m ⁴
Second moment of Area (minor), I _{min}	① 8.28e-5	- 1.09e-4	m ⁴
Section modulus (major), Z _{max}	① 0.00366	- 0.0046	m ³
Structural			
Mass per unit length, m/l	① 132	- 167	kg/m
Bending stiffness (major), E _I max	① 2.33e8	- 2.97e8	N.m ²
Bending stiffness (minor), E _I min	① 1.74e7	- 2.29e7	N.m ²
Failure moment (major), Y _Z max	① 1.24e6	- 1.56e6	N.m

A selection of attributes are shown



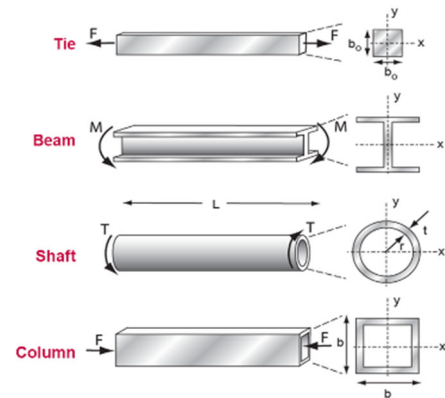
“major” refers to the bending properties in the most favourable direction of the beam. We can see that steels (Y.S. 355) occupy the middle of the chart and that some woods, added for comparison, perform surprisingly well (planks of softwood or laminated beams, marked with stars). It is possible to use a limit stage to remove unsuitable sections in screening but difficult to make a decision about which one has the best performance, using this chart. The shape factor, representing structural efficiency, must be taken into account.

Structural efficiency is obtained by combining material with macroscopic shape. While a material itself can be considered to have properties, it has no intrinsic shape. A structure, or a structural section, is a material made into a shape. Shape factors are measures of the efficiency of material usage for a specific load case, such as bending.



3. Shape factors

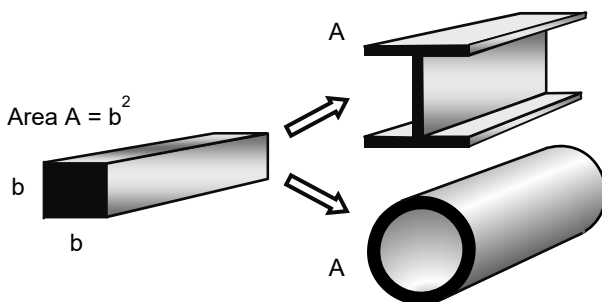
Mechanical loads can be divided into different types; those that exert torque, axial or flexural (bending) loads as discussed above. Typically, one of these dominate in the component, which is also linked to the names of common structural elements; *Shafts* carry torque, *Ties* carry tensile loads while *Columns* carry compressive axial loads and *Beams* bending moments. If we focus on beams in bending, for example, hollow box shapes or I-sections are more efficient than square cross-sections. The reason is, simply, that the material is distributed to places where it contributes more to the stiffness or strength.



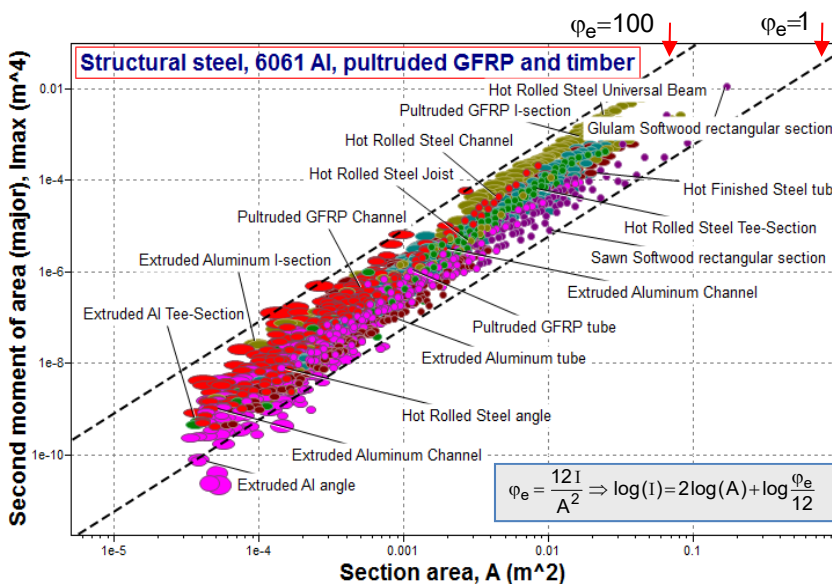
The parameter that reflects the relevant distribution of material in the cross-section is the second moment of the area, $I = \int y^2 b(y) dy$, where y is the distance from the neutral line to the area element representing the location of the material. For a solid square beam, this is: $I_o = \frac{b^4}{12} = \frac{A^2}{12}$, where A is the cross-section area.

This represents the reference shape factor, $\varphi_e=1$. The shape factor for stiffness in bending of any structural section is defined as the ratio: $\varphi_e = \frac{S}{S_o} = \frac{EI}{EI_o} = 12 \frac{I}{A^2}$, where S is called the bending stiffness (see next page).

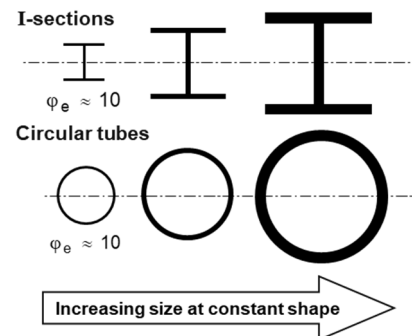
Note that the relevant material-shape combination is $E \cdot I$ for both bending stiffness (S) and deflection (δ). This coincides with the parameters for critical buckling load in compression, familiar from Euler theory [1]. There is an analogous situation for strength-limited design, where the relevant material-shape combination is $\sigma_y \cdot Z$ and the onset of plasticity failure is: $\varphi_f = \frac{F_f}{F_{fo}} = \frac{\sigma_y Z}{\sigma_y Z_o} = 6 \frac{Z}{A^{3/2}}$, F_f being failure force.



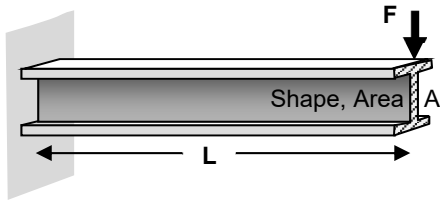
It turns out that a solid circular beam of the same cross-section area as the square beam has nearly the same shape factor, $\varphi_e=0.96$. If the same cross-section area is used in a hollow tube or an I-beam, instead of a square beam, it is around 10 times more efficient in bending stiffness. By using the definition of φ_e and the values of I and A for the individual sections in the data-table, it is possible to plot an overview of shape factors [2].



In practice, the shape factors range from around 1 up to around 100. Note that the size of the section (A) does not affect the shape factor if scaled proportionally.



4. Revised Performance Index for Shaped Beams



In order to derive a performance index that includes both material and shape, we can use an I-bar in bending. Using stiffness-limited design, we assume a lower limit of the bending stiffness (S^*) as a constraint and take as an objective to minimize mass.

Function: Beam in bending, Area and Shape free

Constraint: Bending stiffness $> S^*$

Then, at the limit*: $S = \frac{F}{\delta} = \frac{CEI}{L^3}$

Some useful definitions

A = cross-section area

F = point force

δ = deflection

S = stiffness (F/δ)

E = Young's modulus

I = second moment of area

C = constant (here, 3)

This beam: $\delta = FL^3/CEI = 12FL^3/CEA$

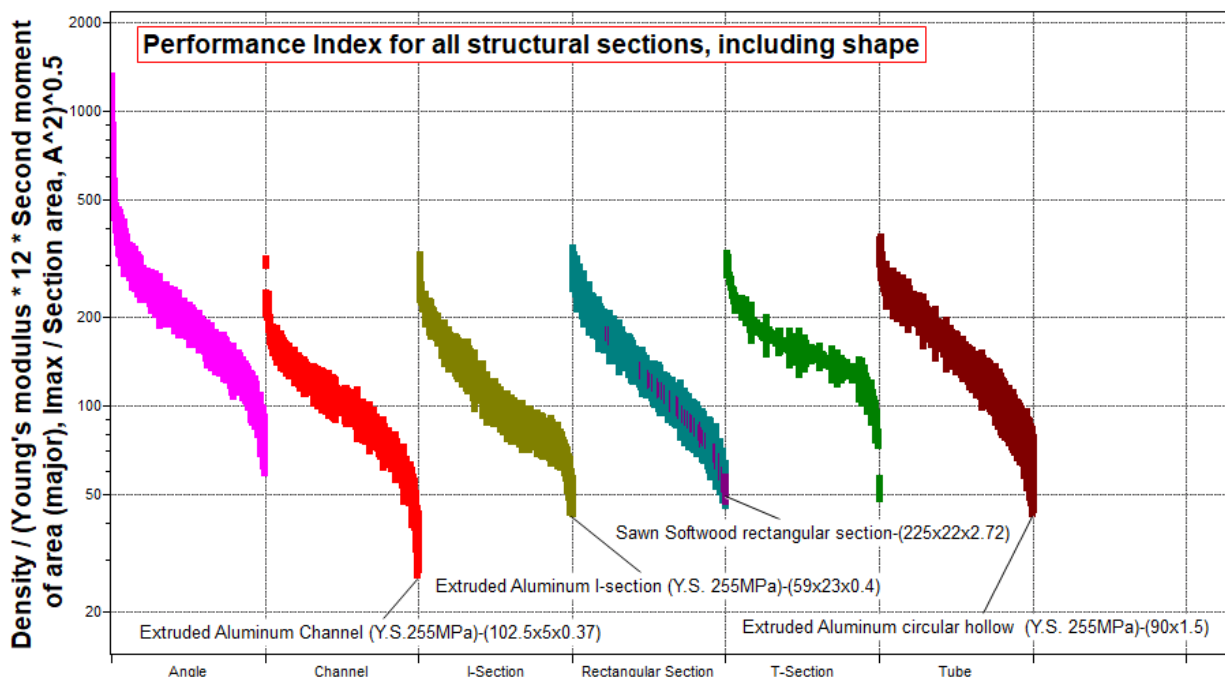
Since: $\varphi_e = 12 \frac{I}{A^2}$, from the previous page, we have: $A = \left(\frac{12I}{\varphi_e} \right)^{1/2}$

Which can be substituted into the objective to eliminate the area but retain the influence of the shape.

Objective: Minimize mass, $m = AL\rho$

If the mass is to be minimized: $m = \left(\frac{12SL^5}{c} \right)^{1/2} \left(\frac{\rho}{(\varphi_e E)^{1/2}} \right)$, then $M = \frac{\rho}{(\varphi_e E)^{1/2}}$ is the performance index to minimize.

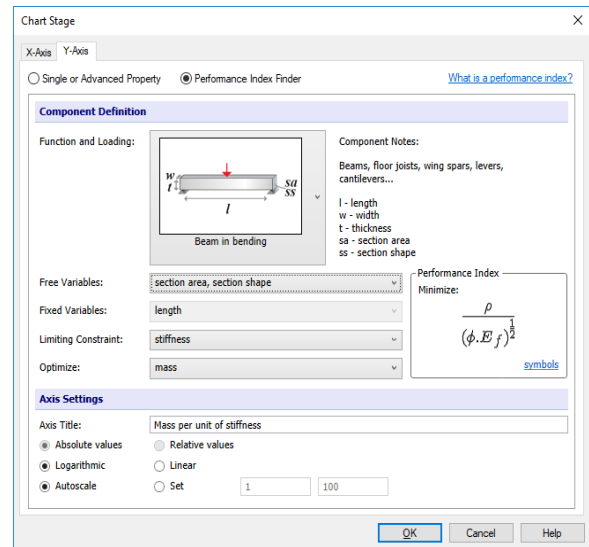
Now, the shape factor is not one of the explicit attributes in the structural section data-table, so it becomes somewhat indirect to express the actual shape factor for each individual record. However, using the same trick as in the chart on the previous page and the advanced option in the *Chart* stage, we can enter an expression for the full revised performance index to minimize. The results, pertaining to the structural section data-table, are shown below, by section type. Extruded Al and softwood planks are performing the best in this overview.



5. Material Selection for Optimal Shape

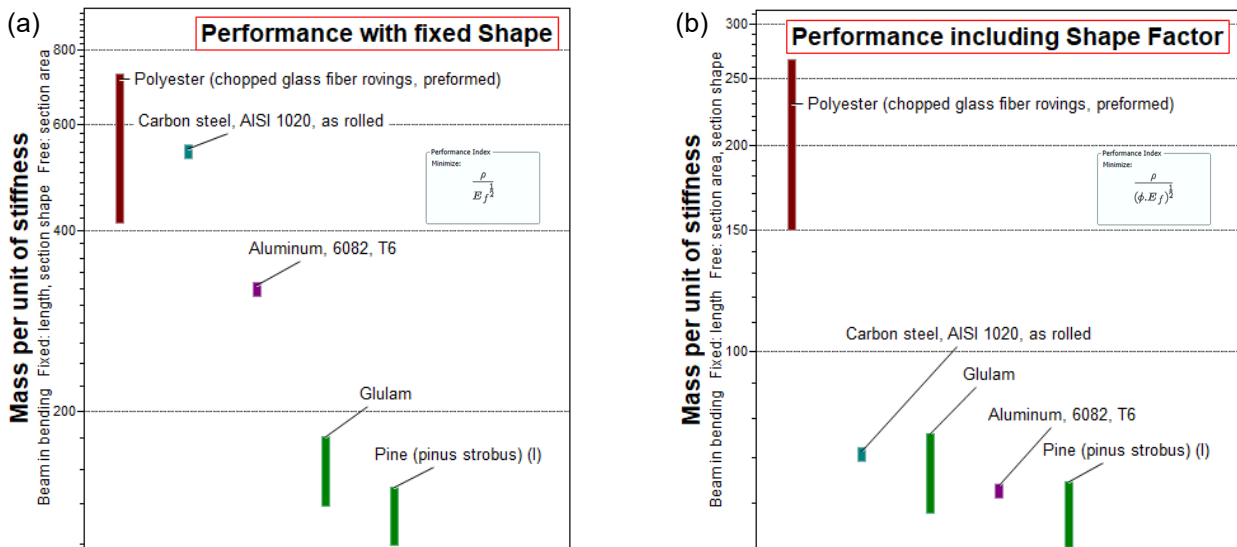
The shape factor can now be utilized to select structural sections, taking into account material and the mechanical efficiency, like in the chart above. In the MaterialUniverse of CES EduPack, however, selection is done by using an optimal elastic shape factor, Φ , listed in the datasheet of properties. The shape factor is not considered a material property but, in our context, it provides an approximate value for the *maximum shape factor* for each material before localized buckling (relevant for a beam in bending). The slenderness the shape, the larger Φ_e is, but there is a limit - make it too thin and the flanges or tube wall will buckle - there is thus a maximum shape factor for each material that depends on its material properties.

This theoretical limit is based on: $\Phi \sim \sqrt{\frac{E}{\sigma_y}}$.



The amended performance index (embedded in CES EduPack via *Learn*) to minimize mass for a beam in bending with stiffness-limited design is: $\rho / (\Phi \cdot E_f)^{1/2}$, where E_f represents the stiffness in bending, or flexural modulus, and Φ is short for Φ_e . This can be plotted into a material property chart, enabling regular visual material selection to get top candidates. The specific shape dimensions, however, remain to be optimized.

The consideration of shape does influence the ranking of the materials significantly in a way that is made apparent in the charts below. The performance index finder in Level 3 has been used to plot the objectives for the cases of the free design parameter being (a) only section area and (b) section shape and section area:



The section length is considered fixed and the minimum mass for a stiffness-limited design is desired. Results from the MaterialUniverse of the five structural section materials show that Pine (softwood) gives the best performance for a beam in bending in both cases. Laminated wood is also performing well whereas GFRP is not very suitable for this type of load. Al and steel show dramatic relative improvements in performance if the cross-section is optimized, since they can be made into I-beams and other very efficient hollow shapes.

Material (from ref 2)	Steels	Aluminum alloys	GFRP and CFRP	Unreinforced polymers	Wood
Max Φ_e (Φ in CES)	65	44	39	12	5
Max Φ_f	13	10	9	5	3

6. Finite Element Simulations

While the property charts can help find the best material options for a light and stiff beam loaded in bending, it does not tell us which dimensions are needed (free design parameters) or the resulting deflection for the different candidates which may be critical in design. The property chart also relies on approximate analytical expressions for the second moments of area, rather than a detailed simulation taking into account the actual geometry of the cross-section. Some of these shortcomings can be addressed by taking top material candidates as input to computer simulations using a Finite Element (FE) approach. This is an approximate numerical method that is widely used and important in engineering and design, including structural mechanics.

In the context of structural sections, if the dimensions are known, FE simulations can estimate the response to external load, for example, elastic deformation. This can be done efficiently and with a high degree of accuracy based on a CAD model of the mechanical object. The solution to the equations depends on *material properties*, such as stiffness, strength, Poisson's ratio etc as well as *geometry*, both which will have to be entered as user inputs. It can be difficult and time-consuming, although not impossible with shape/topology optimization, to find an optimal geometry. It is more difficult to select the best combination of material properties and shape, though. What you are looking for is a structural section that can sustain the maximum internal stress (von Mises) within the stated maximum deflection due to the load (including safety factor, of course). Hence, starting with a small set of well-defined material candidates makes the design process more effective.

7. Reality check

If there are existing numerical limits on the elastic deformation (deflection) of the structural section, there are two ways to connect these to required material properties. The first option is to use approximate analytical expressions via classical equations strength/mechanics of materials (e.g., Roark's Formulas for Stress and Strain) [3]. This allows you to calculate estimated requirements (limits) for strength and stiffness for a specific geometry and load situation, which is a stepping-stone for finding suitable materials (screening). It would not tell you, however, which is the best material (ranking). Material properties and structural data needs to be entered by the user or parametrized in some way. The equations can be worked out manually, or using a software tool, such as the Engineering Solver, embedded in the R&D software CES Selector as shown to the right. This tool supports screening of material options. Given the geometry of the beam and the design parameters (load conditions) it estimates the required material properties and displays the shape factor. This allows you to try different shapes and match this with sufficiently stiff and strong materials from the database.

Engineering Solver

Beam in Bending [Change situation](#)

Estimates the minimum strength, stiffness and shape factor values required for a beam with the specified geometry and load conditions.

Assumptions:

- Material is homogeneous and exhibits the same stiffness in tension & compression
- The beam is nominally straight, with uniform cross section
- Beam is long in proportion to its depth. Minimum length/depth ratio varies as follows:
 - Metal beams with compact section = 8
 - Beams with relatively thin webs = 15
 - Rectangular timber beams = 24

Geometry

Cross-section: I-section

Breadth (b): 180 mm

Depth (d): 140 mm

Thickness (t): 20 mm

Web thickness (tw): 20 mm

Length (l): 5 m

Design parameters

Load conditions: Cantilever End load

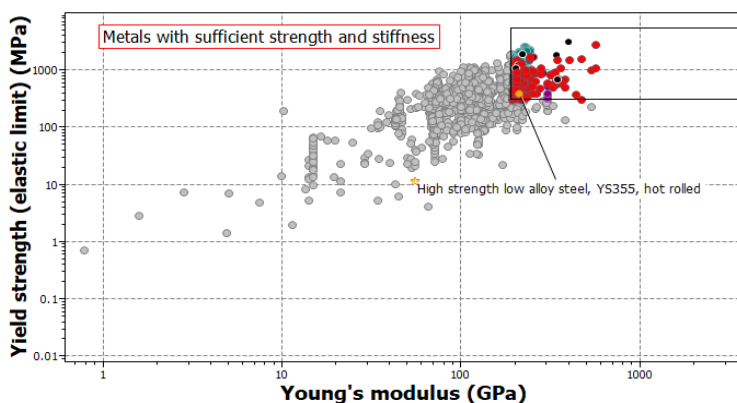
Load: 10000 N

Safety factor: 4.5

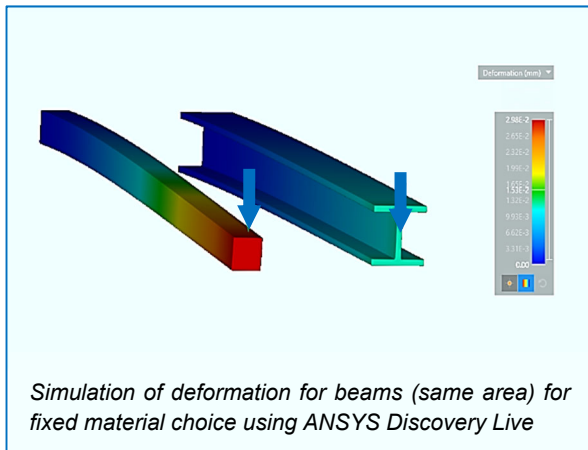
Maximum deflection: 200 mm

Results

Young's modulus	Yield strength	Shape Factor
184 GPa	398 MPa	6.11



The chart to the left and the example above illustrate how mechanical requirements can be tested and met for geometry and design parameters of a suitable candidate. It is screening, rather than ranking of material options, since a performance index is not involved. There is still a need to find optimal materials. Hence, CES EduPack or CES Selector are very useful tools.



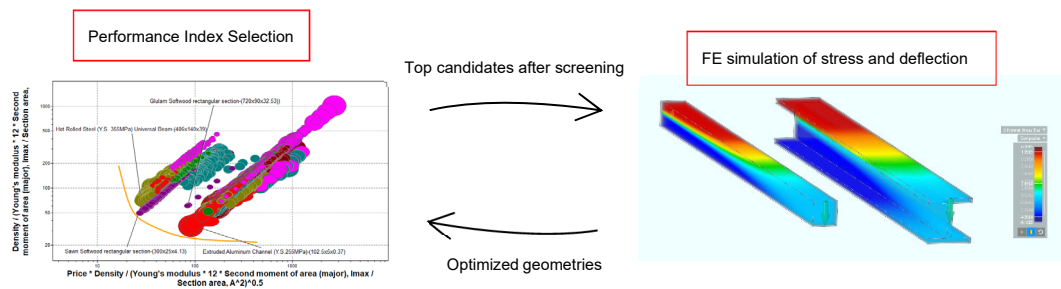
The second option is to solve the governing differential equations for the actual geometry and load case by using numerical computer simulations, such as the FE Method. This can estimate the deflection caused by certain load, or map the stress within the geometry caused by this load, provided the necessary material properties are given. It represents a standard approach when it comes to computer aided engineering. Regardless of which option is pursued, the mutual benefits of a good material selection and geometry design optimization is obvious. For the design engineer, screening based on material properties, ranking by material-shape performance index and FE simulations to fix dimensions are complementary.

8. Combining Material Selection with FE Simulation

CES EduPack is a materials selection tool with materials data to provide a design aid for structural sections. The dedicated data-table in the advanced databases and the Architecture level 2 database can be used to compare and explore options and make basic decisions about what material and type of structural section to choose. In this case study, we have discussed the connections between material and shape to derive a revised performance index making it possible to visually select optimal material for structural sections.

The systematic material selection methodology generates significant synergy with structural simulations. Using CES EduPack to screen and select top material candidates in combination with Finite Element simulation to determine geometry, engineers and designers can reduce the number of material candidates to consider when optimizing structural design. This is done by screening on constraints, such as durability, cost or eco-properties, followed by ranking using the shape-dependent performance index before finding the best design.

Furthermore, one aspect that Finite Element simulations can bring back to selection charts and trade off curves made by CES EduPack in an educational context, is to help determine the free design parameters (thickness, area, etc.) for the candidates with similar values for the Performance Index. This aids the understanding of consequences of design decisions.



References

1. Text: "Materials Selection in Mechanical Design", 5th Edition by M.F. Ashby, Butterworth Heinemann, Oxford, 2016. Chapter 10-11.
2. Many figures and storyline: Text: "**Materials: engineering, science, processing and design**" 4th edition by M.F. Ashby, H.R. Shercliff and D. Cebon, Butterworth Heinemann, Oxford, 2019, Chapters 5 and 7.
3. See, for example, Roark's Formulas for Stress and Strain). W. C. Young and R. G. Budynas, 7 ed. (McGraw-Hill, 2002).