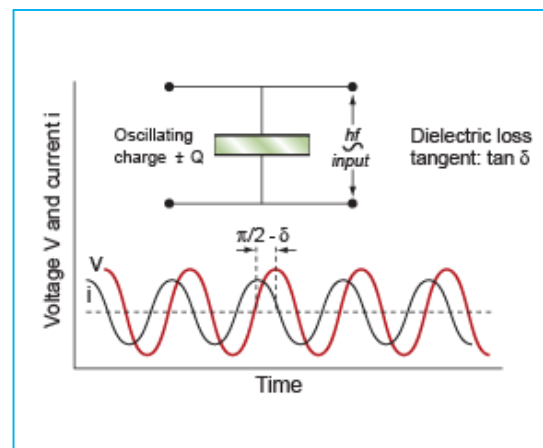
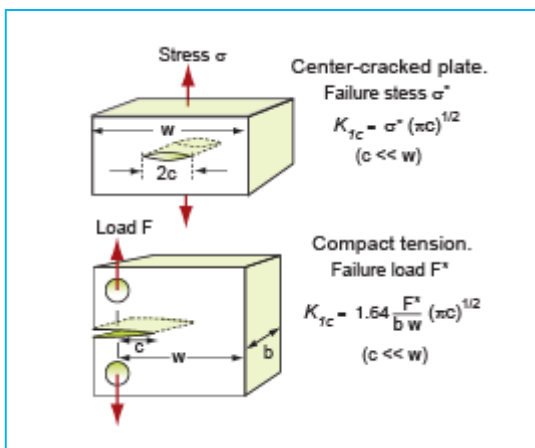
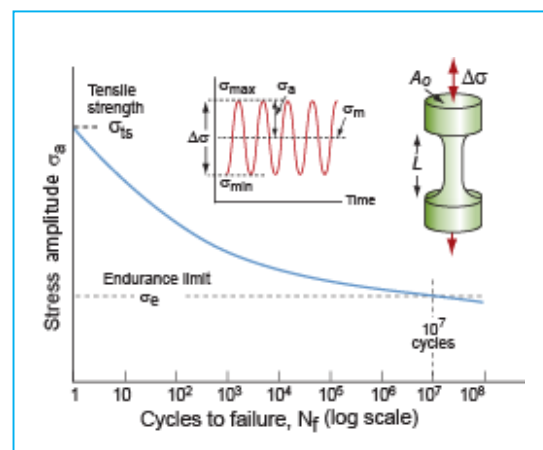
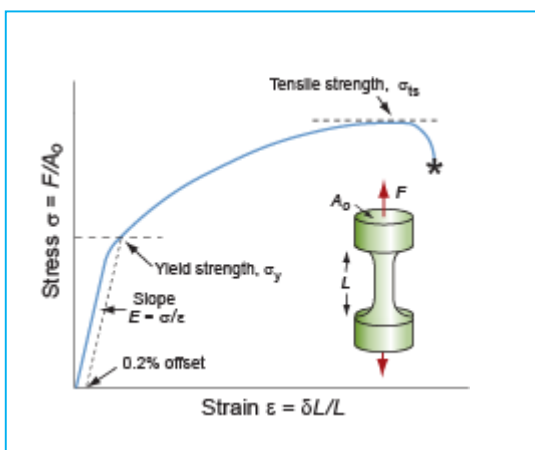


# Material Property Definitions – a summary

Mike Ashby  
Engineering Department and Granta Design  
Cambridge, UK  
4<sup>th</sup> Edition, January 2016



## Physical constants in SI units

Physical constant	Value in SI units
Absolute zero temperature	-273.2 °C
Acceleration due to gravity, g	9.807 m/s <sup>2</sup>
Avogadro's number, N <sub>A</sub>	6.022 x 10 <sup>23</sup> -
Base of natural logarithms, e	2.718 -
Boltzmann's constant, k	1.381 x 10 <sup>-23</sup> J/K
Faraday's constant F	9.648 x 10 <sup>4</sup> C/mol
Gas constant, $\bar{R}$	8.314 J/mol/K
Permeability of vacuum, $\mu_0$	1.257 x 10 <sup>-6</sup> H/m
Permittivity of vacuum, $\epsilon_0$	8.854 x 10 <sup>-12</sup> F/m
Planck's constant, h	6.626 x 10 <sup>-34</sup> J/s
Velocity of light in vacuum, c	2.998 x 10 <sup>8</sup> m/s
Volume of perfect gas at STP	22.41 x 10 <sup>-3</sup> m <sup>3</sup> /mol

## Conversion of units, general

Quantity	Imperial unit	SI unit
Angle, $\theta$	1 rad	57.30°
Density, $\rho$	1 lb/ft <sup>3</sup>	16.03 kg/m <sup>3</sup>
Diffusion coefficient, D	1 cm <sup>2</sup> /s	1.0 x 10 <sup>-4</sup> m <sup>2</sup> /s
Energy, U	See inside back cover	
Force, F	1 kgf	9.807 N
	1 lbf	4.448 N
	1 dyne	1.0 x 10 <sup>-5</sup> N
Length, $\ell$	1 ft	304.8 mm
	1 inch	25.40 mm
	1 Å	0.1 nm
Mass, M	1 tonne	1000 kg
	1 short ton	908 kg
	1 long ton	1107 kg
	1 lb mass	0.454 kg
Power, P	See inside back cover	
Stress, $\sigma$	See inside back cover	
Specific heat, Cp	1 cal/gal.°C	4.188 kJ/kg.°C
	Btu/lb.°F	4.187 kJ/kg.°C
Stress intensity, K <sub>1c</sub>	1 ksi $\sqrt{\text{in}}$	1.10 MN/m <sup>3/2</sup>
Surface energy $\gamma$	1 erg/cm <sup>2</sup>	1 mJ/m <sup>2</sup>
Temperature, T	1°F	0.556K
Thermal conductivity $\lambda$	1 cal/s.cm.°C	418.8 W/m.°C
	1 Btu/h.ft.°F	1.731 W/m.°C
Volume, V	1 Imperial gall	4.546 x 10 <sup>-3</sup> m <sup>3</sup>
	1 US gall	3.785 x 10 <sup>-3</sup> m <sup>3</sup>
Viscosity, $\eta$	1 poise	0.1 N.s/m <sup>2</sup>
	1 lb ft.s	0.1517 N.s/m <sup>2</sup>

# Material property definitions

## - a summary

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Engineering Department and Granta Design  
Cambridge  
January 2016

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# Material property definitions

## 1 Introduction and synopsis

Materials, one might say, are the food of design. This booklet presents the menu: the materials shopping list. A successful product – one that performs well, is good value for money and gives pleasure to the user – uses the best materials for the job, and fully exploits their potential and characteristics. Brings out their flavor, so to speak.

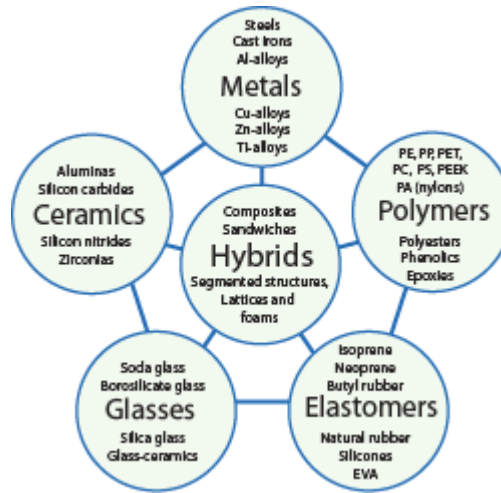
The families of materials – metals, polymers, ceramics, and so forth – are introduced in Section 2. What do we need to know about them if we are to design with them? That is the subject of Section 3, in which distinctions are drawn between various types of materials information. But it is not, in the end, a *material* that we seek; it is a certain *profile of properties* – the one that best meets the needs of the design. Properties are the currency of the materials world. They are the bargaining chips – the way you trade off one material against another. The properties important in thermo-mechanical design are defined briefly in Section 4. Values for these properties can be found in the companion booklet “Material Property Data for Engineering Materials”, 3<sup>rd</sup> edition, Granta Design, 2015.

## 2 The families of engineering materials

It is conventional to classify the materials of engineering into the six broad families shown in Figure 1: metals, polymers, elastomers, ceramics, glasses and hybrids. The members of a family have certain features in common: similar properties, similar processing routes, and, often, similar applications.

**Metals** are stiff – they have relatively high elastic moduli. About 91 of the 118 elements of the periodic table are metals. Most, when pure, are soft and easily deformed. They can be made strong by alloying and by mechanical and heat treatment, but they remain ductile, allowing them to be formed by deformation processes. Certain high-strength alloys (spring steel, for instance) have ductilities as low as 1%, but even this is enough to ensure that the material yields before it fractures and that fracture, when it occurs, is of a tough, ductile type. Partly because of their ductility, metals are prey to fatigue and of all the classes of material, they are the least resistant to corrosion.

**Ceramics**, too, have high moduli, but, unlike metals, they are brittle. Their "strength" in tension means the brittle fracture strength; in compression it is the brittle crushing strength, which is about 15 times greater. And because ceramics have no ductility, they have a low tolerance for stress concentrations (like holes or cracks) or for high contact stresses (at clamping points, for instance). Ductile materials accommodate stress concentrations by deforming in a way that redistributes the load more evenly, and because of this, they can be used under static loads within a small margin of their yield strength. Ceramics cannot. Brittle materials always have a wide scatter in strength and the strength itself depends on the volume of material under load and the time over which it is applied. So design with ceramics is more difficult than design with metals. Despite this, they have attractive features. They are stiff, hard and abrasion-resistant (hence their use for bearings and cutting tools); they retain their strength to high temperatures (they are used as thermal-barrier coatings on aircraft turbine blades); and they resist corrosion well (thus their use as toilets, basins and kitchen work-surfaces).



**Figure 1.** The menu of engineering materials. The basic families of metals, ceramics, glasses, polymers and elastomers can be combined in various geometries to create hybrids.

**Glasses** are non-crystalline (“amorphous”) solids. The most common are the soda-lime and borosilicate glasses familiar as bottles and ovenware, but there are many more. Metals, too, can be made non-crystalline by cooling them sufficiently quickly. The lack of crystal structure suppresses plasticity, so, like ceramics, metallic glasses are hard, brittle and vulnerable to stress concentrations.

**Polymers** are at the other end of the spectrum. They have moduli that are low, roughly 50 times less than those of metals, but they can be strong – nearly as strong as metals. A consequence of this is that elastic deflections can be large. They creep, even at room temperature, meaning that a polymer component under load may, with time, acquire a permanent set. And their properties depend on temperature so that a polymer that is tough and flexible at 20°C may be brittle at the 4°C of a household refrigerator, yet creep rapidly at the 100°C of boiling water. Few have useful strength above 200°C. Some polymers are mainly crystalline, some are amorphous (non-crystalline), some a mix of crystalline and amorphous – the transparent ones are amorphous. If these aspects are allowed-for in the design, the advantages of polymers can be exploited. And there are many. When combinations of properties, such as strength-per-unit-weight, are important, polymers can compete with metals. They are easy to shape. Complicated parts performing several functions can be molded from a polymer in a single operation. The large elastic deflections allow the design of polymer components that snap together, making assembly fast and cheap. And by accurately sizing the mold and pre-coloring the polymer, no finishing operations are needed. Polymers resist corrosion (paints, for instance, are polymers) and have low coefficients of friction. Good design exploits these properties.

**Elastomers** are long-chain polymers above their glass-transition temperature,  $T_g$ . The covalent bonds that link the units of the polymer chain remain intact, but the weaker Van der Waals and hydrogen bonds that, below  $T_g$ , bind the chains to each other, have melted. This gives elastomers unique properties: elastic moduli as low as  $10^{-3}$  GPa ( $10^5$  time less than that typical of metals) that increase with temperature (all other solids show a decrease), and enormous elastic extension. Their properties differ so much from those of other solids that special tests have evolved to characterize them. This creates a problem: if we wish to select materials by prescribing a desired attribute profile, then a prerequisite is a set of attributes common to all materials. To overcome this, we use a common set of properties in the early stages of design, estimating approximate values for anomalies like elastomers. Specialized attributes, representative of one family only, are for use in the later stages.

**Hybrids** are combinations of two or more materials in a pre-determined configuration and scale (Figure 1). The best of them combine the attractive properties of the other families of materials while avoiding some of their drawbacks. The family of hybrids includes fiber and particulate composites,

sandwich structures, lattice structures, foams, cables and laminates, and almost all the materials of nature – wood, bone, skin, leaf. Fiber-reinforced composites are, of course, the most familiar. Most of those at present available to the engineer have a polymer matrix reinforced by fibers of glass, carbon or Kevlar (an aramid). They are light, stiff and strong, and they can be tough. They, and other hybrids using a polymer as one component, cannot be used above 250°C because the polymer softens, but at room temperature their performance can be outstanding. Hybrid components are expensive and they are relatively difficult to form and join. So, despite their attractive properties, the designer will use them only when the added performance justifies the added cost. Many hybrids are lighter than metals of the same strength. Today's emphasis on high performance and fuel efficiency provides increasing drivers for their use.

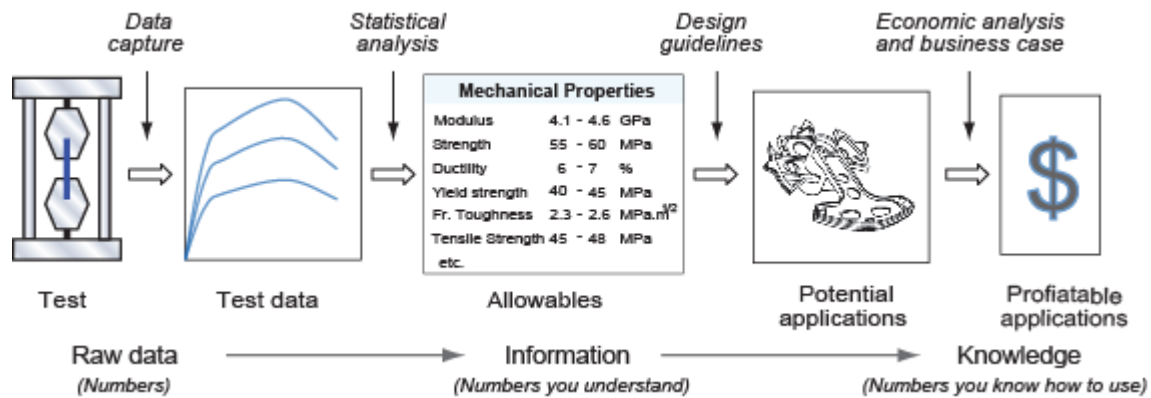
These, then, are the material families. What do we need to know about them?

### 3 Materials information for design

The engineer, in selecting materials for a developing design, needs data for their properties. Engineers are often conservative in their choice, reluctant to consider materials with which they are unfamiliar, and with good reason. Data for the old, well-tried materials are established, reliable, and easily-found. Data for newer, emerging, materials may be incomplete or untrustworthy. Yet innovation is often made possible by new materials. So it is important to know how to judge data quality.

If you're going to design something, what sort of materials information do you need? Figure 2 draws relevant distinctions. On the left a material is tested and the *data* are captured. But this raw data – unqualified numbers – are, for our purposes, useless. To make data useful requires statistical analysis. What is the mean value of the property when measured on a large batch of samples? What is the standard deviation? Given this it is possible to calculate *allowables*, values of properties that, with a given certainty (say, one part in 10<sup>6</sup>) can be guaranteed. Material texts generally present test data; by contrast, data in most engineering handbooks are allowables. One can think of data with known precision and provenance as *information*. It can generally be reported as tables of numbers, as yes/no statements or as rankings: that is, it can be *structured*. Many attributes that can be structured are common to all materials; all, for instance, have a density, an elastic modulus, a strength, a thermal conductivity. Structured information can be stored in a database and – since all materials have values – it is the starting point for selecting between them.

This is a step forward, but it is not enough. To design with a material, you need to know its real character, its strengths and its weaknesses. How do you shape it? How join it? Who has used it before and for what? Did it fail? Why? This information exists in handbooks, is documented as design guidelines, and is reported in failure analyses and case studies. It consists largely as text, graphs and images, and while certain bits of it may be available for one material, for another they may not. It is messier, but it is essential in reaching a final selection. We refer to this supporting information as *documentation*. The image and text on the right of the ABS cover- record are examples of documentation.



**Figure 2.** Types of material information.

There is more. Material uses are subject to standards and codes. These rarely refer to a single material but to classes or subclasses. For a material to be used in contact with food or drugs, it must carry FDA approval or the equivalent. Metals and composites for use in US military aircraft must have Military Specification Approval. To qualify for best-practice design for the environment, material usage must conform to ISO 14040 guidelines. And so forth. This, too, is a form of documentation. The ensemble of information about a material, structured and unstructured, constitutes *knowledge*.

There is yet more (Figure 2, right-hand side). To succeed and profitable in the market place, a product must be economically viable and compete successfully, in terms of performance, consumer appeal and cost, with the competition. All of these depend on material choice and the way the material is processed. Much can be said about this, but not here; for now, the focus is on structured data and documentation.

That's the essential background. Now for the properties themselves.

## 4 Material properties and their units

Each material can be thought of as having a set of attributes or properties. The combination that characterizes a given material is its *property-profile*. Property profiles are assembled by systematic testing. In this section we scan the nature of the tests and the definition and units of the properties. Units are given here in the SI system. Conversion factors to other systems are printed on the inside front and back cover of this booklet.

### *General properties.*

Property	Symbol and Units
Density	$\rho$ (kg/m <sup>3</sup> )
Price	$C_m$ (\$/kg)

**Density.** The density,  $\rho$  (units: kg/m<sup>3</sup>) is the mass per unit volume. We measure it today as Archimedes did: by weighing in air and in a fluid of known density.

**Price.** The price,  $C_m$ , (units: \$/kg) of materials spans a wide range. Some cost as little as \$0.2/kg, others more than \$1000/kg. Prices, of course, fluctuate, and they depend on the quantity you want and on your status as a "preferred customer" with your chosen vendor. Despite this uncertainty, it is useful to have an approximate price, useful in the early stages of material selection.

## Mechanical properties

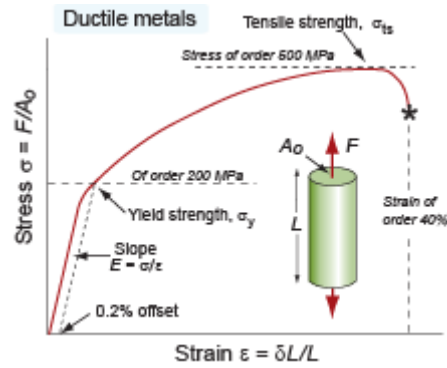
Property	Symbol and Units
Elastic moduli (Young's, Shear, Bulk)	$E, G, K$ (GPa)
Yield strength	$\sigma_y$ (MPa)
Tensile (ultimate) strength	$\sigma_{ts}$ (MPa)
Compressive strength	$\sigma_c$ (MPa)
Failure strength	$\sigma_f$ (MPa)
Hardness	$H$ (Vickers)
Elongation	$\varepsilon$ (%)
Fatigue endurance limit	$\sigma_e$ (MPa)
Fracture toughness	$K_{Ic}$ (MPa.m <sup>1/2</sup> )
Toughness	$G_{Ic}$ (kJ/m <sup>2</sup> )
Loss coefficient (damping capacity)	$\eta$ (%)
Wear rate (Archard) constant	$K_A$ MPa <sup>-1</sup>

**Elastic modulus.** The elastic modulus  $E$  (units: GPa or GN/m<sup>2</sup>) is the slope of the initial, linear-elastic, part of the stress-strain curve (Figure 3). Young's modulus,  $E$ , describes response to tensile or compressive loading, the shear modulus,  $G$ , describes response to shear loading and the bulk modulus,  $K$ , response to hydrostatic pressure. Poisson's ratio,  $\nu$ , is the negative of the ratio of the lateral strain,  $\varepsilon_2$ , to the axial strain,  $\varepsilon_1$ , in axial loading:

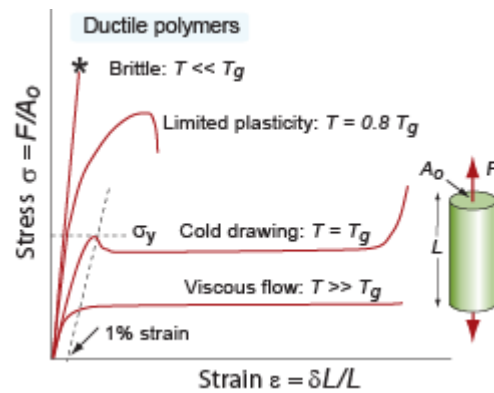
$$\nu = -\frac{\varepsilon_2}{\varepsilon_1}$$

In reality, moduli measured as slopes of stress-strain curves are inaccurate, often low by a factor of 2 or more, because of contributions to the strain from anelasticity, creep and other factors. Accurate moduli are measured dynamically: by exciting the natural vibrations of a beam or wire, or by measuring the velocity of sound waves in the material.

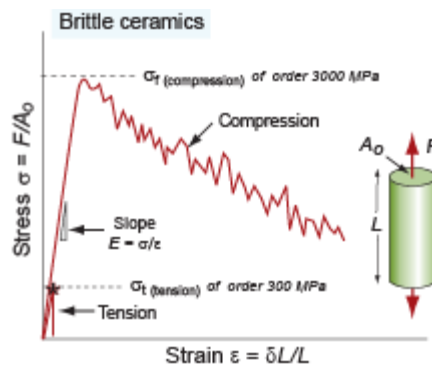




**Figure 3.** The stress-strain curve for a metal, showing the modulus,  $E$ , the 0.2% yield strength,  $\sigma_y$ , and the ultimate strength  $\sigma_{ts}$ .



**Figure 4.** Stress-strain curves for a polymer, below, at and above its glass transition temperature,  $T_g$ .



**Figure 5.** Stress-strain curves for a ceramic in tension and in compression. The compressive strength  $\sigma_c$  is 10 to 15 times greater than the tensile strength  $\sigma_t$ .

In an isotropic material, the moduli are related in the following ways:

$$E = \frac{3G}{1+G/3K}; \quad G = \frac{E}{2(1+\nu)}; \quad K = \frac{E}{3(1-2\nu)} \quad (1)$$

Commonly  $\nu \approx 1/3$  when

$$G \approx \frac{3}{8}E \quad \text{and} \quad K \approx E \quad (2,a)$$

Elastomers are exceptional. For these  $\nu \approx 1/2$  when

$$G \approx \frac{1}{3}E \quad \text{and} \quad K \gg E \quad (2,b)$$

Data sources list values for all four moduli.

**Estimating moduli.** Young's modulus  $E$  for copper is 124 GPa.; its Poisson's ratio  $\nu$  is 0.345. What is its shear modulus,  $G$  ?

**Answer.** Inserting the values for  $E$  and  $\nu$  in the central equation (1) gives  $G = 46.1$  GPa. The measured value is 45.6 GPa, a difference of only 1%.

**Strength.** The strength,  $\sigma_f$ , (units: MPa or MN/m<sup>2</sup>) of a solid requires careful definition. For metals, we identify  $\sigma_f$  with the 0.2% offset yield strength  $\sigma_y$  (Figure 3), that is, the stress at which the stress-strain curve for axial loading deviates by a strain of 0.2% from the linear-elastic line. It is the same in tension and compression. For polymers,  $\sigma_f$  is identified as the stress at which the stress-strain curve becomes markedly non-linear, at a strain, typically, of 1% (Figure 4). This may be caused by shear-yielding: the irreversible slipping of molecular chains; or it may be caused by crazing: the formation of low density, crack-like volumes that scatter light, making the polymer look white. Polymers are a little stronger ( $\approx 20\%$ ) in compression than in tension.

Strength, for ceramics and glasses, depends strongly on the mode of loading (Figure 5). In tension, "strength" means the fracture strength,  $\sigma_t$ . In compression it means the crushing strength  $\sigma_c$ , which is much greater; typically

$$\sigma_c = 10 \text{ to } 15 \sigma_t \quad (3)$$

**Modulus of rupture.** When a material is difficult to grip, as is a ceramic, its strength can be measured in bending. The flexural strength or modulus of rupture,  $\sigma_{flex}$  (units: MPa) is the maximum surface stress in a bent beam at the instant of failure (Figure 6). One might expect this to be the same as the strength measured in tension, but for ceramics it is greater by a factor of about 1.3, because the volume subjected to this maximum stress is small and the probability of a large flaw lying in it is small also; in simple tension all flaws see the same stress.

The strength of a composite is best defined by a set deviation from linear-elastic behavior; often an offset of 0.5% is taken. Composites that contain fibers, including natural composites such as wood, are a little weaker (up to 30%) in compression than tension because fibers buckle.

Strength, then, depends on material class and on mode of loading. Other modes of loading are possible: shear, for instance. Yield under multi-axial loads is related to that in simple tension by a *yield function*. For metals, the Von Mises' yield function is a good description:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_f^2 \quad (4)$$

where  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  are the principal stresses, positive when tensile;  $\sigma_1$ , by convention, is the greatest or most positive,  $\sigma_3$  the smallest or least positive. For polymers the yield function is modified to include the effect of pressure

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_f^2 \left(1 + \frac{\beta p}{K}\right)^2 \quad (5)$$

where  $K$  is the bulk modulus of the polymer,  $\beta \approx 2$  is a numerical coefficient that characterizes the pressure dependence of the flow strength and the pressure  $p$  is defined by

$$p = -\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

For ceramics, a Coulomb flow law is used:

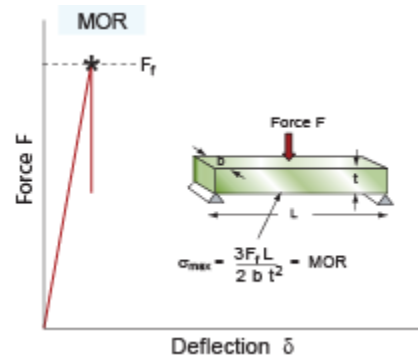
$$\sigma_1 - B\sigma_2 = C \quad (6)$$

where B and C are constants.

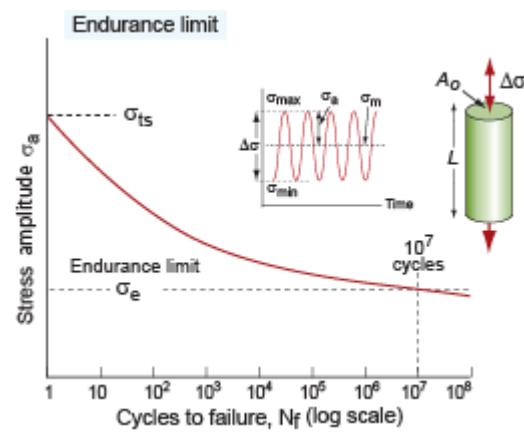
**Using yield functions.** A metal pipe of radius  $r$  and wall thickness  $t$  carries an internal pressure  $p$ . The pressure generates a circumferential wall-stress of  $\sigma_1 = pr/t$  an axial wall-stress  $\sigma_2 = pr/2t$ . At what pressure will the pipe first yield?

**Answer.** Setting  $\sigma_2 = \sigma_1/2$ ,  $\sigma_3 = 0$  and  $\sigma_f = \sigma_y$  in equation 4 gives the yield condition  $\sigma_1 = (2/\sqrt{3})\sigma_y$ . Thus the pressure  $p^*$  that just causes first yield is  $p^* = \frac{2}{\sqrt{3}} \frac{t}{r} \sigma_y$ .

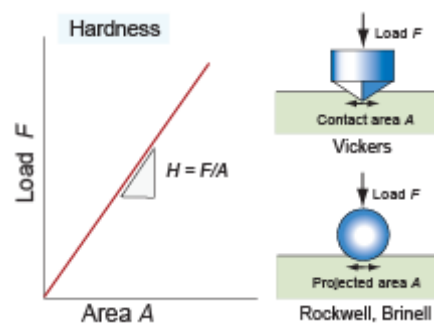
**Tensile strength.** The tensile (or ultimate) strength  $\sigma_{ts}$  (units: MPa) is the nominal stress at which a round bar of the material, loaded in tension, separates (Figure 3). For brittle solids – ceramics, glasses and brittle polymers – it is the same as the failure strength in tension. For metals, ductile polymers and most composites, it is greater than the yield strength,  $\sigma_y$ , by a factor of between 1.1 and 3 because of work hardening or, in the case of composites, load transfer to the reinforcement.



**Figure 6.** The modulus-of-rupture (MOR) is the surface stress at failure in bending. It is equal to, or slightly larger than the failure stress in tension.



**Figure 7.** The endurance limit,  $\sigma_e$ , is the cyclic stress that causes failure in  $N_f = 10^7$  cycles.



**Figure 8.** Hardness is measured as the load  $F$  divided by the projected area of contact,  $A$ , when a diamond-shaped indenter is forced into the surface.

**Endurance limit.** Cyclic loading can cause a crack to nucleate and grow in a material, culminating in fatigue failure. For many materials there exists a fatigue or endurance limit,  $\sigma_e$  (units: MPa), illustrated by the  $\Delta\sigma - N_f$  curve of Figure 7. It is the stress amplitude  $\Delta\sigma$  below which fracture does not occur, or occurs only after a very large number ( $N_f > 10^7$ ) of cycles.

**Hardness.** Tensile and compression tests are not always convenient: a large sample is needed and the test destroys it. The hardness test gives an approximate, non-destructive, measure of the strength. The hardness,  $H$ , (SI units: MPa) of a material is measured by pressing a pointed diamond or hardened steel ball into the surface of the material (Figure 8). The hardness is defined as the indenter force divided by the projected area of the indent. It is related to the quantity we have defined as  $\sigma_f$  by

$$H \approx 3\sigma_f \quad (7)$$

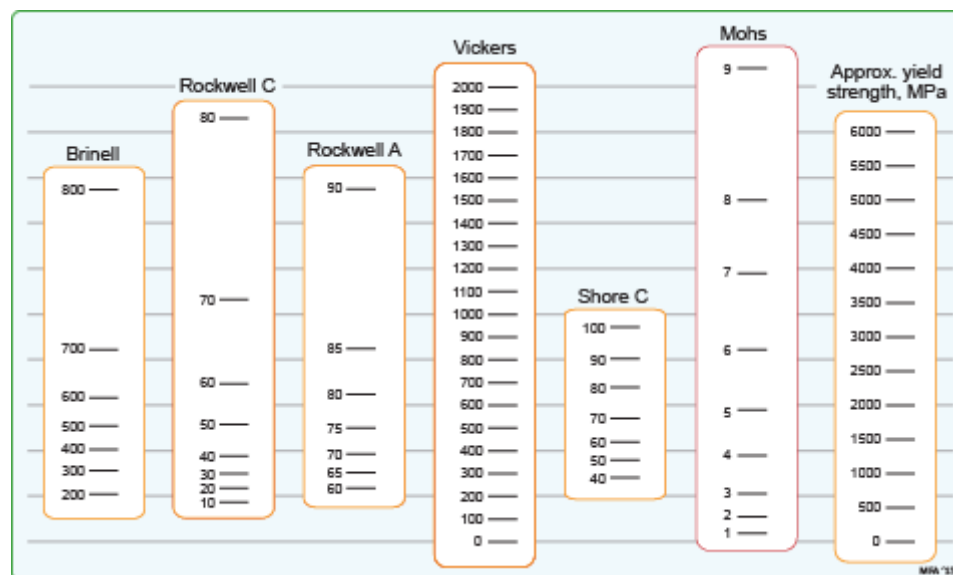
This, in the SI system, has units of MPa. Hardness is commonly reported in a bewildering array of other units, the most common of which is the Vickers  $H_v$  scale with units of  $\text{kg}/\text{mm}^2$ . It is related to  $H$  in the units used here by

$$H_v = \frac{H}{10}$$

A conversion chart for five hardness scales, relating them to yield strength, appears in Figure 9.

**Strength from hardness.** A steel has a hardness of 50 on the Rockwell C scale. Approximately what is its Vickers hardness and yield strength?

**Answer.** The chart of Figure 9 shows that the Vickers hardness corresponding to a Rockwell C value of 50 is approximately  $H_v = 500$  and the yield strength is approximately 1700 MPa.



**Figure 9.** Commonly used scales of hardness related to each other and to the yield strength.

**Toughness and Fracture toughness.** The toughness,  $G_{Ic}$ , (units: kJ/m<sup>2</sup>) and the fracture toughness,  $K_{Ic}$ , (units: MPa m<sup>1/2</sup> or MN/m<sup>1/2</sup>) measure the resistance of a material to the propagation of a crack. The fracture toughness is measured by loading a sample containing a deliberately-introduced crack of length  $2c$  (Figure 10), recording the tensile stress  $\sigma^*$  at which the crack propagates. The quantity  $K_{Ic}$  is then calculated from

$$K_{Ic} = Y \sigma^* \sqrt{\pi c} \quad (8)$$

and the toughness from

$$G_{Ic} = \frac{K_{Ic}^2}{E(1+\nu)} \quad (9)$$

where  $Y$  is a geometric factor, near unity, that depends on details of the sample geometry,  $E$  is Young's modulus and  $\nu$  is Poisson's ratio. Measured in this way  $K_{Ic}$  and  $G_{Ic}$  have well defined values for brittle materials (ceramics, glasses, and many polymers). In ductile materials a plastic zone develops at the crack tip, introducing new features into the way in which cracks propagate that necessitate more involved characterization. Values for  $K_{Ic}$  and  $G_{Ic}$  are, nonetheless, cited, and are useful as a way of ranking materials.

**Using fracture toughness.** A glass floor panel contains micro-cracks up to 2 microns in length. Glass has a fracture toughness of  $K_{Ic} = 0.6 \text{ MPa}\cdot\text{m}^{1/2}$ . When the panel is walked upon, stresses as high as 30 MPa appear in it. Is it safe?

**Answer.** The stress required to make a 2 micron crack (so  $c = 10^{-6} \text{ m}$ ) propagate in glass with a fracture toughness of  $K_{Ic} = 0.6 \text{ MPa}\cdot\text{m}^{1/2}$ , using equation 8 with  $Y = 1$ , is

$$\sigma_c = K_{Ic} / \sqrt{\pi c} = 339 \text{ MPa}.$$

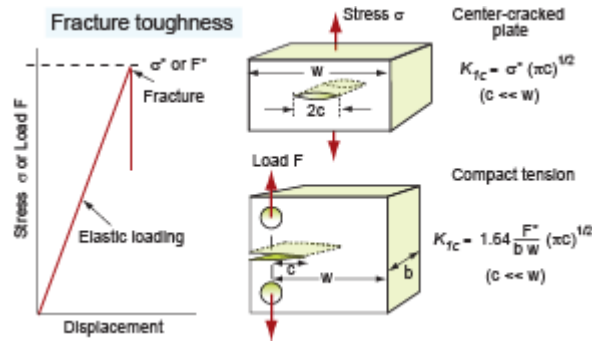
The panel is safe.

**Loss coefficient.** The loss-coefficient,  $\eta$  (a dimensionless quantity), measures the degree to which a material dissipates vibrational energy (Figure 11). If a material is loaded elastically to a stress  $\sigma_{max}$ , it stores an elastic energy

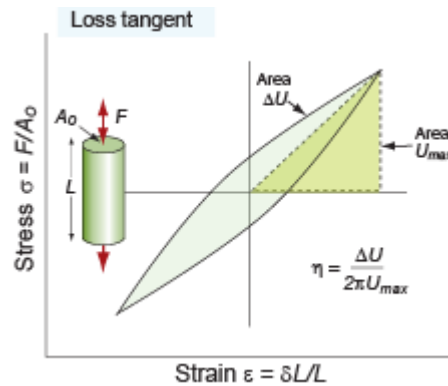
$$U = \int_0^{\sigma_{max}} \sigma d\varepsilon \approx \frac{1}{2} \frac{\sigma_{max}^2}{E}$$

per unit volume. If it is loaded and then unloaded, it dissipates an energy

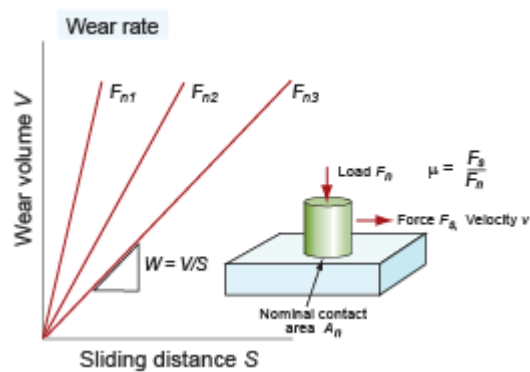
$$\Delta U = \oint \sigma d\varepsilon$$



**Figure 10.** The fracture toughness,  $K_{Ic}$ , measures the resistance to the propagation of a crack. The test specimen containing a crack of length  $2c$  fails at a stress  $\sigma^*$ . The fracture toughness is then  $K_{Ic} = Y\sigma^* \sqrt{\pi c}$  where  $Y$  is a constant near unity.



**Figure 11.** The loss tangent  $\eta$  measures the fractional energy dissipated in a stress-strain cycle.



**Figure 12.** Wear is the loss of material from surfaces when they slide. The wear resistance is measured by the Archard wear constant  $K_A$ , defined in the text.

The loss coefficient is

$$\eta = \frac{\Delta U}{2\pi U_{max}} \quad (10)$$

where  $U_{max}$  is the stored elastic energy at peak stress. The value of  $\eta$  usually depends on the time-scale or frequency of cycling.

Other measures of damping include the **specific damping capacity**,  $D = \Delta U / U$ , the **log decrement**,  $\Delta$  (the log of the ratio of successive amplitudes of natural vibrations), the **phase-lag**,  $\delta$ , between stress and strain, and the **"Q"-factor** or **resonance factor**,  $Q$ . When damping is small ( $\eta < 0.01$ ) these measures are related by

$$\eta = \frac{D}{2\pi} = \frac{\Delta}{\pi} = \tan \delta = \frac{1}{Q} \quad (11)$$

but when damping is large, they are no longer equivalent.

**Using loss coefficients.** A bell with a natural frequency of  $f = 1000$  Hz is made of a material with a loss coefficient  $\eta = 0.01$ . For how long will it ring after being struck? If the material is replaced by one of low damping with  $\eta = 10^{-4}$  how long will it ring? (Assume the ring has ended when the amplitude of oscillation  $A$  has fallen to one hundredth of its initial value).

**Answer.** Let  $A$  and  $A + dA$  be amplitudes of successive cycles ( $dA$  is negative). Then

$$\text{Log}\left(\frac{A}{A+dA}\right) = \Delta = \pi\eta \quad \text{from which} \quad \frac{dA}{A} = \frac{1}{10^{\pi\eta}} - 1$$

Integrating over  $n$  cycles gives  $\ln \frac{A}{A_0} = \left( \frac{1}{10^{\pi\eta}} - 1 \right) n$  where  $A_0$  is the initial amplitude. When  $A$  has fallen to  $0.01A_0$ , the term  $\ln(A/A_0) = -4.6$ , giving  $n = 4.6 \left( \frac{10^{\pi\eta}}{10^{\pi\eta} - 1} \right)$ . Thus a bell with  $\eta = 0.01$  will ring for  $n = 66$  cycles, giving a time  $n/f = 66$  milliseconds. A bell with  $\eta = 10^{-4}$  will ring for  $n = 6400$  cycles and a time of  $n/f = 6.4$  seconds.

**Archard wear constant.** Wear, the loss of material when surfaces slide against each other, is a multi-body problem. Nevertheless, it can, to a degree, be quantified. When solids slide (Figure 12) the volume of material lost from one surface, per unit distance slid, is called the wear rate,  $W$  (units:  $m^2$ ). The wear resistance of the surface is characterized by the Archard wear constant,  $K_A$  (units:  $1/\text{MPa}$ ) defined by the equation

$$\frac{W}{A} = K_A P \quad (16)$$

where  $A$  is the area of the slider surface and  $P$  the normal force pressing it onto the other surface.



**Calculating wear.** A steel slider oscillates on a dry steel substrate at frequency  $f = 0.2$  Hz and an amplitude  $a = 2$  mm under a normal pressure  $P = 2$  MPa. The Archard wear constant for steel on steel is  $K_A = 3 \times 10^{-8}$  (MPa) $^{-1}$ . By how much will the surface of the slider have been reduced in thickness after a time  $t = 100$  hours?

**Answer.** The distance slid in 100 hours is  $d = 4 a f t$  m. The thickness  $x$  removed from the slider over the time  $t = 3.6 \times 10^5$  s is

$$x = \frac{\text{Volume removed}}{\text{Area } A} = 4 a f t K_A P = 3.5 \times 10^{-5} \text{ m} = 36 \mu\text{m}$$

### Thermal properties

Property	Symbol and Units
Melting point	$T_m$ (°C or K)
Glass temperature	$T_g$ (°C or K)
Maximum service temperature	$T_{max}$ (°C or K)
Minimum service temperature	$T_{min}$ (°C or K)
Thermal conductivity	$\lambda$ (W/m.K)
Specific heat	$C_p$ (J/kg.K)
Thermal expansion coefficient	$\alpha$ (K $^{-1}$ )
Thermal shock resistance	$\Delta T_s$ (°C or K)

**Melting point and glass temperature.** Two temperatures, the melting temperature,  $T_m$ , and the glass temperature,  $T_g$  (units for both: K or C) are fundamental because they relate directly to the strength of the bonds in the solid. Crystalline solids have a sharp melting point,  $T_m$ . Non-crystalline solids do not; the temperature  $T_g$  characterizes the transition from true solid to very viscous liquid.

**Maximum and minimum service temperature.** It is helpful, in engineering design, to define two further temperatures: the maximum and minimum service temperature  $T_{max}$  and  $T_{min}$  (both: K or C). The first tells us the highest temperature at which the material can reasonably be used without oxidation, chemical change or excessive creep becoming a problem. The second is the temperature below which the material becomes brittle or otherwise unsafe to use.

**Heat capacity.** It costs energy to heat a material up. The heat capacity or specific heat (units J/kg.K) is energy to heat 1 kg of a material by 1 K. The measurement is usually made at constant pressure (atmospheric pressure) so it is given the symbol  $C_p$ . When dealing with gases, it is more usual to measure the heat capacity at constant volume (symbol  $C_v$ ) and for gases this differs from  $C_p$ . For solids the difference is so slight that it can be ignored, and we shall do so here. The heat capacity is measured by calorimetry

(Figure 13), which is also the standard way of measuring the glass temperature  $T_g$ . A measured quantity of energy (here, electrical energy) is pumped into a sample of material of known mass. The temperature rise is measured allowing the energy/kg.K to be calculated. Real calorimeters are more elaborate than this, but the principle is the same.

**Using specific heat.** How much energy is required to heat a 100 mm cube of copper from room temperature (20 °C) to its melting point?

**Answer.** Data for melting point,  $T_m$ , specific heat,  $C_p$  and density  $\rho$  for copper are  $T_m = 1082$  °C,  $C_p = 380$  J/kg.K and  $\rho = 8930$  kg/m<sup>3</sup>. The mass of copper in the cube is  $\rho V = 8.93$  kg. The energy to heat it through  $\Delta T = 1062$  °C is

$$\rho V C_p \Delta T = 3.6 \text{ MJ.}$$

(The energy in a liter of gasoline is 35 MJ).

**Thermal conductivity.** The rate at which heat is conducted through a solid at steady state (meaning that the temperature profile does not change with time) is measured by the thermal conductivity,  $\lambda$  (units: W/m.K). Figure 14 shows how it is measured: by recording the heat flux  $q$  (W/m<sup>2</sup>) flowing through the material from a surface at higher temperature  $T_1$  to a lower one at  $T_2$  separated by a distance  $X$ . The conductivity is calculated from Fourier's law:

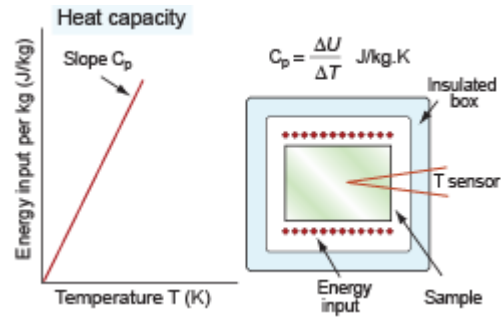
$$q = -\lambda \frac{dT}{dX} = \lambda \frac{(T_1 - T_2)}{X} \quad (12)$$

The measurement is not, in practice, easy (particularly for materials with low conductivities), but reliable data are now generally available.

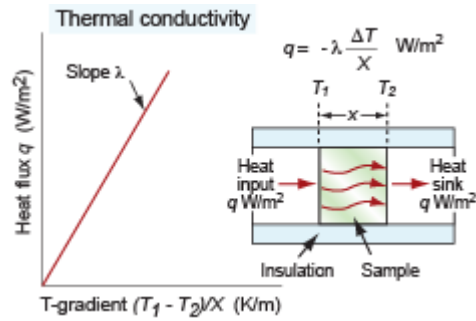
**Steady state heat flow.** A heat exchanger has an exchange-area of  $A = 0.5$  m<sup>2</sup>. It passes heat from a fluid at temperature  $T_1 = 100$  °C to a second fluid at  $T_2 = 20$  °C. The exchange wall is made of copper sheet (thermal conductivity  $\lambda = 350$  W/m.K) with a thickness  $X = 2$  mm. How much energy flows from one fluid to the other in one hour?

**Answer.** The temperature gradient  $dT/dX = 80/0.002 = 40,000$  °C/m. The total energy  $Q$  that passes across the area  $A$  over a time  $t = 3600$  seconds is

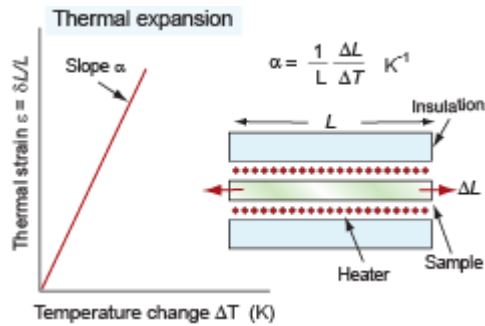
$$Q = Atq = At\lambda \frac{dT}{dX} = 2.5 \times 10^{10} \text{ J} = 25 \text{ GJ}$$



**Figure 13.** The heat capacity – the energy to raise the temperature of one kg of material by one degree Celsius.



**Figure 14.** The thermal conductivity  $\lambda$  measures the flux of heat driven by a temperature gradient  $dT/dX$ .



**Figure 15.** The linear-thermal expansion coefficient  $\alpha$  measures the change in length, per unit length, when the sample is heated.

**Thermal diffusivity.** When heat flow is transient, the flux depends instead on the thermal diffusivity,  $a$  (units:  $\text{m}^2/\text{s}$ ), defined by

$$a = \frac{\lambda}{\rho C_p} \quad (13)$$

where  $\rho$  is the density and  $C_p$  is the heat capacity. The thermal diffusivity can be measured directly by measuring the decay of a temperature pulse when a heat source, applied to the material, is switched off; or it can be calculated from  $\lambda$ , via the last equation. The distance  $x$  heat diffuses in a time  $t$  is approximately

$$x \approx \sqrt{2\alpha t} \quad (14)$$

**Transient heat flow.** You pour boiling water into a tea-glass with a wall thickness  $x = 3 \text{ mm}$ . How many seconds have you got to carry it to the table before it becomes too hot to hold? (The thermal conductivity of glass  $\lambda = 1.1 \text{ W/m.K}$ , its density is  $\rho = 2450 \text{ kg/m}^3$  and its heat capacity  $C_p = 800 \text{ J/kg.K}$ .)

**Answer.** Inserting the data into equation 13 gives a thermal diffusivity for glass of  $a = 5.6 \times 10^{-7} \text{ m}^2/\text{s}$ . Inserting this into equation 14 gives the approximate time

$$t \approx \frac{x^2}{2a} = 8 \text{ seconds.}$$

**Thermal expansion coefficient.** Most materials expand when they are heated (Figure 15). The thermal strain per degree of temperature change is measured by the linear thermal-expansion coefficient,  $\alpha$  (units:  $\text{K}^{-1}$  or, more conveniently, as “microstrain/ $^\circ\text{C}$ ” or  $10^{-6} \text{ }^\circ\text{C}^{-1}$ ). If the material is thermally isotropic, the volume expansion, per degree, is  $3\alpha$ . If it is anisotropic, two or more coefficients are required, and the volume expansion becomes the sum of the principal thermal strains.

**Thermal stress.** An aluminum pipe is rigidly clamped to the face of a concrete building. On a hot day the face of the building in direct sun rises to  $80 \text{ }^\circ\text{C}$ , and because the expansion of the aluminum is greater than that of concrete, stress appears in it. What is the value of the stress if the original clamping was done on a day when the temperature was  $20 \text{ }^\circ\text{C}$ ?

**Answer.** The expansion coefficient of aluminum is  $\alpha = 22.5 \times 10^{-6} / ^\circ\text{C}$ , that of concrete is  $\alpha = 9 \times 10^{-6} / ^\circ\text{C}$ . The aluminum pipe is rigidly clamped, so the difference in thermal strain  $\Delta\alpha \Delta T = 13.5 \times 10^{-6} \times 60 = 8.1 \times 10^{-4}$ . This has to be accommodated by elastic compression of the aluminum (modulus  $E = 75 \text{ GPa}$ ), giving a stress  $\Delta\alpha \Delta T E = 61 \text{ MPa}$ . This is enough to cause a soft aluminum to yield.

**Thermal shock resistance.** The thermal shock resistance  $\Delta T_s$  (units: K or °C) is the maximum temperature difference through which a material can be quenched suddenly without damage. The thermal shock resistance and the **creep resistance** are important in high-temperature design. Creep is the slow, time-dependent deformation that occurs when materials are loaded above about  $\frac{1}{3}T_m$  or  $\frac{2}{3}T_g$ . Design against creep is a specialized subject. Here we rely instead on avoiding the use of a material above its maximum service temperature,  $T_{max}$ , or, for polymers, the “heat deflection temperature”.

### Electrical properties

Property	Symbol and Units
Electrical resistivity	$\rho_e$ ( $\Omega.m$ or $\mu\Omega.cm$ )
Dielectric constant	$\epsilon_r$ (–)
Breakdown potential	$V_b$ ( $10^6 V/m$ )
Power factor	$P$ (–)
Piezoelectric charge coefficient	$d_{33}$ pC/N or pm/V
Piezoelectric voltage coefficient	$g_{33}$ mV.m/N
Pyroelectric coefficient	$\gamma$ $\mu C/m^2.K$
Curie temperature (ferroelectric)	$T_c$ °C

**Electrical resistivity.** The electrical resistivity,  $\rho_e$  (SI units  $\Omega.m$ , or, commonly  $\mu\Omega.cm$ ) is the resistance of a unit cube with unit potential difference between a pair of its faces (Figure 16). It has an immense range, from a little more than  $10^{-8}$  in units of  $\Omega.m$  for good conductors (equivalent to  $1 \mu\Omega.cm$ ) to more than  $10^{16} \Omega.m$  ( $10^{24} \mu\Omega.cm$ ) for the best insulators. The electrical conductivity  $\kappa_e$  (units Siemens per meter ( $S/m$  or  $(\Omega.m)^{-1}$ )) is simply the reciprocal of the resistivity.

**Resistivity and resistance.** Tungsten has a conductivity of  $\kappa_e = 8.3 \times 10^6$  Siemens. What is the resistance of a tungsten wire of radius  $r = 100$  microns in diameter and length  $L = 1$  m?

**Answer.** The resistivity of tungsten  $\rho_e = 1/\kappa_e = 1.2 \times 10^{-7} \Omega.m$ . The resistance  $R$  of the wire is

$$R = \rho_e \frac{L}{\pi r^2} = 3.8 \Omega$$

**Dielectric constant.** When an insulator (or *dielectric*) is placed in an electric field, it becomes polarized and charges appear on its surfaces that tend to screen the interior from the electric field. The tendency to polarize is measured by the dielectric constant,  $\epsilon_r$ , a dimensionless quantity (Figure 17). Its value for free space and, for practical purposes, for most gasses, is 1. Most insulators have values between 2 and 30, though low density foams approach the value 1 because they are largely air.

What does  $\epsilon_r$  measure? Two conducting plates separated by a dielectric make a capacitor. Capacitors store charge. The charge  $Q$  (units: coulombs) is directly proportional to the voltage difference between the plates,  $V$  (volts):

$$Q = CV \quad (15)$$

where  $C$  (farads) is the capacitance. The capacitance of a parallel plate capacitor of area  $A$ , separated by empty space (or by air) is

$$C = \epsilon_o \frac{A}{t} \quad (16)$$

where  $\epsilon_o$  is the *permittivity of free space* ( $8.85 \times 10^{-12} \text{ F/m}$ , where  $F$  is farads). If the empty space is replaced by a dielectric, capacitance increases because of its polarization. The field created by the polarization opposes the field  $E$ , reducing the voltage difference  $V$  needed to support the charge. Thus the capacity of the condenser is increased to the new value

$$C = \epsilon \frac{A}{t} \quad (17)$$

where  $\epsilon$  is the *permittivity of the dielectric* with the same units as  $\epsilon_o$ . It is usual to cite, not this, but the *relative permittivity* or *dielectric constant*,  $\epsilon_r$ :

$$\epsilon_r = \frac{C_{withdielectric}}{C_{nodielectric}} = \frac{\epsilon}{\epsilon_o} \quad (18)$$

making the capacitance

$$C = \epsilon_r \epsilon_o \frac{A}{t} \quad (19)$$

When charged, the energy stored in a capacitor is

$$\frac{1}{2} QV = \frac{1}{2} CV^2 \quad (20)$$

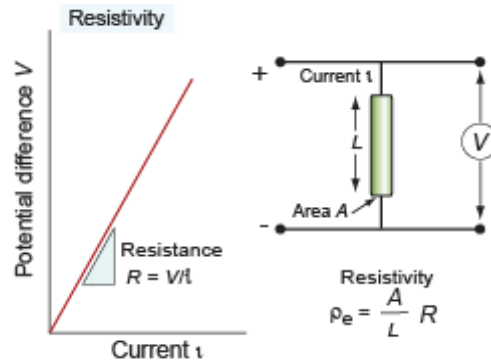
and this can be large: “super-capacitors” with capacitances measured in farads store enough energy to power a hybrid car.

**Stray capacitance.** The time constant  $\tau$  for charging or discharging a capacitor is

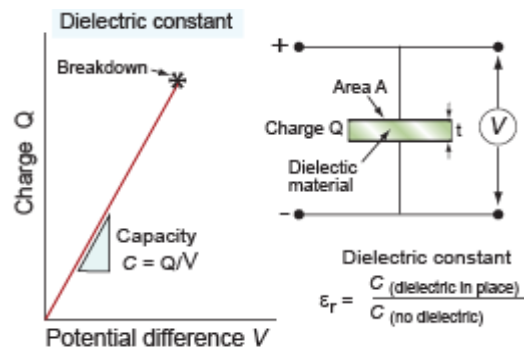
$$\tau = RC.$$

where  $R$  is the resistance of the circuit. That means that stray capacitance in an electronic circuit (capacitance between neighbouring conducting lines or components) slows its response. What material choices minimize this?

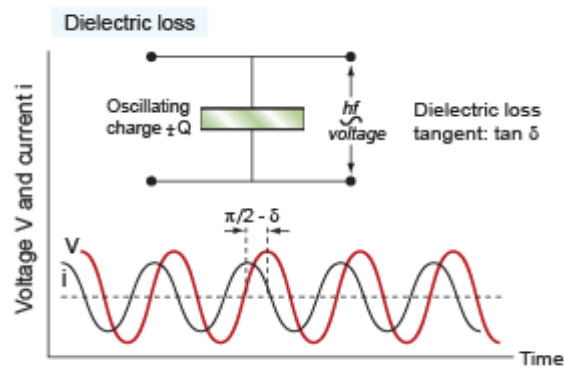
**Answer.** Choosing materials with low resistivity  $\rho_e$  for the conductors (to minimize  $R$ ) and choosing insulators with low dielectric constant  $\epsilon_r$  to separate them (to minimize  $C$ ), minimizes  $\tau$ .



**Figure 16.** Electrical resistivity,  $\rho_e$ , is measured as the potential gradient  $V/L$  divided by the current density,  $i/A$ . It is related to resistance  $R$  by  $\rho_e = AR/L$ .



**Figure 17.** Dielectric constant: a measure of the ability of an insulator to polarize.



**Figure 18.** Dielectric loss, important in dielectric heating, as explained in the text.

**Dielectric loss tangent.** Polarization involves the small displacement of charge (either of electrons or of ions) or of molecules that carry a dipole moment when an electric field is applied to the material. An oscillating field drives the charge between two alternative configurations. This charge-motion is like an electric current that – if there were no losses – would be 90° out of phase with the voltage. In real dielectrics this current dissipates energy, just as a current in a resistor does, giving it a small phase shift,  $\delta$  (Figure 18). The loss tangent,  $\tan \delta$ , also called the dissipation factor,  $D$ , is the tangent of the loss angle. The power factor,  $P_f$ , is the sine of the loss angle. When  $\delta$  is small, as it is for the materials of interest here, all three are essentially equivalent:

$$P_f \approx D \approx \tan \delta \approx \sin \delta \quad (21)$$

More useful, for our purposes, is the *loss factor*  $L$  which is the loss tangent times the dielectric constant:

$$L = \epsilon_r \tan \delta \quad (22)$$

It measures the energy dissipated by a dielectric when in an oscillating field. If you want to select materials to minimize or maximize dielectric loss, then the measure you want is  $L$ .

When a dielectric material is placed in a cyclic electric field of amplitude  $E$  and frequency  $f$ , power  $P$  is dissipated and the field is correspondingly attenuated. The power dissipated per unit volume, (W/m<sup>3</sup>) is

$$P \approx f E^2 \epsilon \tan \delta = f E^2 \epsilon_0 \epsilon_r \tan \delta = f E^2 \epsilon_0 L \quad (23)$$

where, as before,  $\epsilon_r$  is the dielectric constant of the material and  $\tan \delta$  is its loss tangent. This power appears as heat; the higher the frequency or the field strength and the greater the loss factor  $L = \epsilon_r \tan \delta$  the greater are the heating and energy loss. Sometimes this dielectric loss is exploited in processing – for example, in radio frequency welding of polymers.

**Dielectric heating.** A nylon component is placed in a microwave cavity with a field strength  $E = 10^4$  V/m and a frequency  $f = 10^{10}$  Hz for a time  $t = 100$  s. The dielectric loss factor for nylon is  $L = 0.1$ , its density is  $\rho = 1130$  kg/m<sup>3</sup> and its heat capacity is  $C_p = 1650$  J/kg.K. Assuming no heat loss, how hot will the component become?

**Answer.** The heat generated by the field is  $Q = Pt = f E^2 \epsilon_0 L t = 8.85 \times 10^7$  J/m<sup>3</sup>. The heat capacity of nylon per unit volume is  $C_p \rho = 1.86 \times 10^6$  J/m<sup>3</sup>.K. The temperature rise,  $\Delta T$  is

$$\Delta T = \frac{f E^2 \epsilon_0 L t}{C_p \rho} = 47.6 \text{ } ^\circ\text{C}$$

**Breakdown potential.** The breakdown potential (units: MV/m) is the electrical potential gradient at which an insulator breaks down and a damaging surge of current flows through it. It is measured by increasing, at a uniform rate, a 60 Hz alternating potential applied across the faces of a plate of the material until breakdown occurs, typically at a potential gradient between 1 and 100 million volts per meter (Units: MV/m).



**Piezoelectric charge coefficient,  $d_{33}$  (pC/N).** The piezoelectric effect appears in structures that lack a center of symmetry, such as the compounds barium titanate and lead zirconium titanate (PZT). The distortion of a piezoelectric crystal by an applied stress causes positive and negative ions within the crystal to move relative to each other, changing their dipole moment. This causes charge to appear at the faces of the crystal (which becomes polarized) and a potential difference to form across it (Figure 19). The potential difference can be large – large enough to trigger a spark discharge, an effect used in gas and cigarette lighters.

The polarization  $P_3$  that appears when a piezo-electric crystal is compressed is a linear function of the applied stress  $\sigma_3$

$$P_3 = d_{33}\sigma_3 \quad (24)$$

where  $d_{33}$  is the *piezoelectric charge coefficient*.

**Piezoelectric voltage coefficient,  $g_{33}$  (mV.m/N).** The piezoelectric voltage coefficient,  $g_{33}$  relates electric field  $E_3$  to applied stress  $\sigma_3$

$$E_3 = g_{33}\sigma_3 \quad (25)$$

Electric field is related to polarization by

$$E_3 = \frac{1}{\epsilon_r \epsilon_0} P_3 = \frac{d_{33}}{\epsilon_r \epsilon_0} \sigma_3,$$

meaning that

$$g_{33} = \frac{d_{33}}{\epsilon_r \epsilon_0} \quad (26)$$

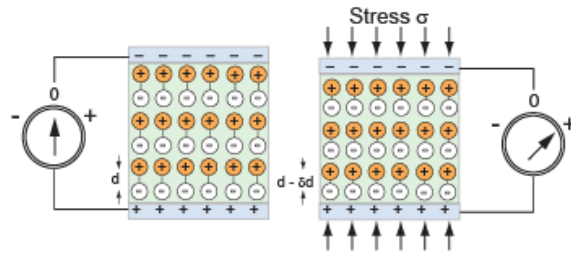
where  $\epsilon_r$  is the relative permittivity (dielectric constant) and  $\epsilon_0$  the permittivity of vacuum ( $8.854 \times 10^{-12}$  F/m).

**Using piezoelectric properties.** A disk of bismuth titanate is subjected to a compressive stress of 2 MPa. How large is the electric field that this stress will generate? Use the mean value of the piezoelectric voltage coefficient  $g_{33}$  to calculate it.

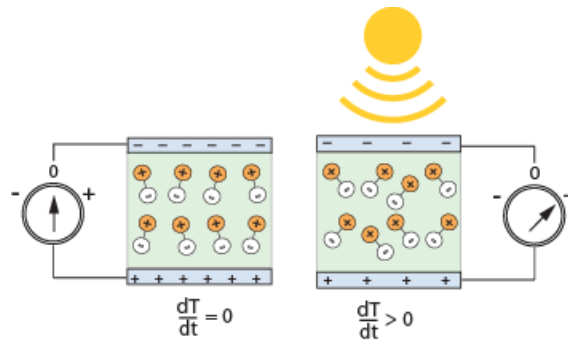
**Answer.** The mean value of the piezoelectric voltage coefficient for bismuth titanate is

$$13.5 \frac{\text{mV.m}}{\text{N}} = 13.5 \times 10^{-3} \frac{\text{V/m}}{\text{N/m}^2}$$

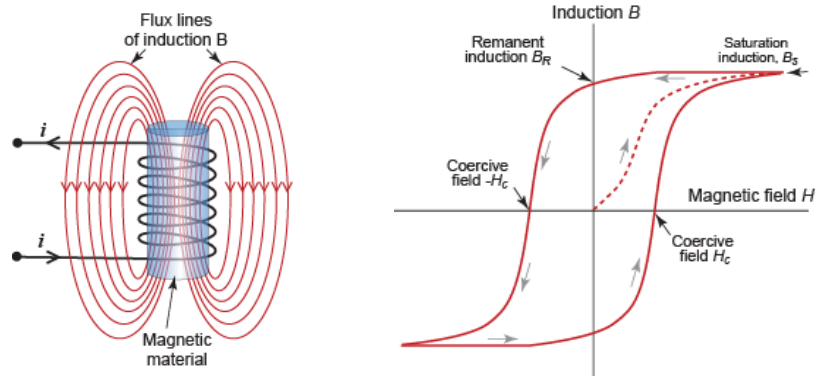
Thus a stress of 2 MPa will create a field of  $13.5 \times 10^{-3} \times 2 \times 10^6 \text{ V/m} = 27,000 \text{ V/m}$



**Figure 19.** The piezoelectric effect



**Figure 20.** The pyroelectric effect



**Figure 21(a).** A magnetic material exposed to a field  $H$  becomes magnetized. The induction.

**Figure 21(b).** The  $B - H$  curve of a ferromagnetic material, showing the important properties.

**Pyroelectric coefficient ( $\mu\text{C}/\text{m}^2\cdot\text{K}$ ).** Below the ferroelectric Curie temperature,  $T_c$ , crystals with a unique direction carry a permanent dipole (even in the absence of an applied field) because positive and negative charges do not cancel along the unique direction. When the temperature is constant the charged faces of the crystal attract opposite charges from the surroundings, screening the charge. But when the temperature changes, the positive and negative charges along the unique direction move relative to one another, changing the polarization as in Figure 20. The effect is used in motion sensors and for infra-red imaging.

The change in polarization  $\Delta P$  caused by a change of temperature  $\Delta T$  is given by

$$\Delta P = \gamma \Delta T \quad (27)$$

Above the Curie temperature  $T_c$  the polarization is lost completely.

### **Magnetic and magnetostrictive properties**

Attribute	Unit
Remanent induction $B_r$	T
Coercive field $H_c$	A/m
Maximum energy product	MJ/m <sup>3</sup>
Max permeability	No unit
Saturation magnetostriction	$\mu\text{strain}$
Curie temperature (magnetic)	°C

**Remanent induction (Tesla) and Coercive field (A/m).** Nearly all materials respond to a magnetic field by becoming magnetized, but most are paramagnetic with a response so faint that it is of no practical use. *Ferromagnetic* and *ferrimagnetic* materials (*ferrites* for short), however, contain atoms that have large magnetic moments and with the ability to spontaneously magnetize – to align their moments in parallel – much as electric dipoles do in ferroelectric materials. These are of real practical use.

When a current  $i$  passes through a long, empty coil of  $n$  turns and length  $L$ , a magnetic field is generated. The magnitude of the field,  $H$ , is given by Ampère's law as

$$H = \frac{ni}{L}$$

and thus has units of amps/meter (A/m). The field induces a *magnetic induction* or *flux density*,  $B$ , which for vacuum or non-magnetic materials is

$$B = \mu_0 H \quad (28)$$

where  $\mu_0$  is the *permeability of vacuum*,  $\mu_0 = 4\pi \times 10^{-7}$  henry/meter (H/m). The units of  $B$  are *tesla*, so a tesla is 1 HA/m<sup>2</sup>.

If the space inside the coil of is filled with a material, as in Figure 21(a), the induction within it changes. This is because its atoms respond to the field by forming little magnetic dipoles. The material acquires a macroscopic dipole moment or *magnetization*,  $M$  (its units are A/m, like  $H$ ). The induction becomes

$$B = \mu_0 (H + M)$$

If the material of the core is ferro-magnetic, the response is a very strong one and it is non-linear. It is usual to rewrite  $B$  in the form

$$B = \mu_R \mu_0 H \quad (29)$$

where  $\mu_R$  is the *relative permeability*. It is dimensionless. The *remanent induction*  $B_R$  is the induction that remains when the field  $H$  is removed. The *coercive field*  $H_C$  is the field required to fully magnetize the material, or (equivalently) to de-magnetize it (Figure 21(b)).

**Saturation magnetostriction ( $\mu$ strain).** Magnetostriction is strain caused by a magnetic field. The material is described as having positive magnetostriction if its length increases with increasing magnetic field and negative magnetostriction if it does the opposite. The *saturation magnetostriction* is the maximum strain that can be induced in a material by applying a magnetic field.

**Using magnetostrictive properties.** A rod of Galfenol (and alloy of gallium and iron, roughly  $\text{Ga}_{0.2}\text{Fe}_{0.8}$ ),  $L = 20$  mm long, is surrounded by a coil of the same length with  $n = 400$  turns. By how much will the rod contract if a current of 0.1 amp is passed through the coil? Use the mean value of the saturation magnetostriction and coercive field for Galfenol to find out.

**Answer.** The field created by a current of 0.1 amp passing through a coil

$$H = \frac{n i}{L} = \frac{400 \times 0.1}{2 \times 10^{-2}} = 2000 \text{ A / m .}$$

The coercive field for Galfenol is 800 A/m. The field created by the coil far exceeds this field, thus saturating the rod and creating a strain of  $150 \times 10^{-6}$ . Thus the rod will contract by

$$\delta L = 150 \times 10^{-6} \times 20 \times 10^{-3} = 3 \times 10^{-6} \text{ m} = 3 \text{ microns}$$

## Optical properties

Property	Symbol and Units
Refractive index	$n$ (–)

All materials allow some passage of light, although for metals it is exceedingly small. The speed of light when in the material,  $v$ , is always less than that in vacuum,  $c$ . A consequence is that a beam of light striking the surface of such a material at an angle of incidence,  $\alpha$ , enters the material at an angle  $\beta$ , the angle of refraction. The **refractive index**,  $n$  (dimensionless) is

$$n = \frac{c}{v} = \frac{\sin \alpha}{\sin \beta} \quad (17)$$

It is related to the dielectric constant,  $\epsilon_r$ , at the same frequency by

$$n \approx \sqrt{\epsilon_r}$$

The refractive index depends on wavelength and thus on the color of the light. The denser the material, and the higher its dielectric constant, the greater is the refractive index. When  $n=1$ , the entire incident intensity enters the material, but when  $n > 1$ , some is reflected. If the surface is smooth and polished, it is reflected as a beam; if rough, it is scattered. The percentage reflected,  $R$ , is related to the refractive index by

$$R = \left( \frac{n-1}{n+1} \right)^2 \times 100 \quad (15)$$

As  $n$  increases, the value of  $R$  approaches 100%.

### **Eco properties**

Property	Symbol and Units
Embodied energy	$H_m$ (MJ/kg)
Carbon footprint	$CO_2$ (kg/kg)

The **embodied energy** (units MJ/kg) is the energy required to extract 1 kg of a material from its ores and feedstock. The associated **CO<sub>2</sub> footprint** (units: kg/kg) is the mass of carbon dioxide released into the atmosphere during the production of 1 kg of material

## **5 Summary and conclusions**

There are six important families of materials for mechanical design: metals, ceramics, glasses, polymers, elastomers, and hybrids that combine the properties of two or more of the others. Within a family there is certain commonality. Ceramics and glasses as a family are hard, brittle and corrosion resistant. Metals are ductile, tough and good thermal and electrical conductors. Polymers are light, easily shaped and electrical insulators. Elastomers have the ability to deform elastically to large strains. That is what makes the classification useful. But in design we wish to escape from the constraints of family, and think, instead, of the material name as an identifier for a certain property-profile. To that end, the properties important in thermo-mechanical design were defined in this booklet.

## 6 Further reading

*Definitions of material properties can be found in numerous general texts on engineering materials, among them those listed here.*

Ashby, M. Shercliff, H. and Cebon, D. (2014) 3<sup>rd</sup> edition, *Materials: Engineering, Science, Processing and Design*, Butterworth-Heinemann, Oxford, UK. ISBN-13: 978-0-08-097773-7, North American Edition: ISBN-13: 978-1-85617-743-6. *(An introductory text introducing ideas that are developed more fully in the present text.)*

Askeland, D.R., Phulé, P.P. and Wright W.J. (2010) *The Science and Engineering of Materials*, 6<sup>th</sup> edition, Thomson, Toronto, Canada. ISBN 9780495296027. *(A well-established materials text that deals well with the science of engineering materials.)*

ASM Handbooks, Volume 8 (2004) "Mechanical testing and evaluation" ASM International, Metals Park, Ohio, USA. *(An on-line, subscription-based resource, detailing testing procedures for metals and ceramics.)*

Budinski, K.G. and Budinski, M.K. (2010) "Engineering materials, properties and selection", 9<sup>th</sup> edition, Prentice Hall, NY, USA. ISBN 978-0-13-712842-6. *(Like Askeland, this is a well-established materials text that deals well with both material properties and processes.)*

Callister, W.D. and Rethwisch, D.G. (2014) *Materials Science and Engineering: An Introduction*, 9<sup>th</sup> edition, John Wiley & Sons, New York, USA. ISBN 978-1-118-54689-5. *(A well-established text taking a science-led approach to the presentation of materials teaching.)*

Dieter, G.E. (1999) "Engineering design, a materials and processing approach", 3<sup>rd</sup> edition, McGraw-Hill, New York, USA. ISBN: 9-780-073-66136-0. *(A well-balanced and respected text focusing on the place of materials and processing in technical design.)*

Farag, M.M. (2013) "Materials and process selection for engineering design", 3<sup>rd</sup> edition, CRC Press, Taylor and Francis, London UK. ISBN 978146656409. *(A Materials-Science approach to the selection of materials)*

Shackelford, J.F. (2014) "Introduction to materials science for engineers", 8<sup>th</sup> edition, Prentice Hall, NJ, USA. ISBN 978-0133789713. *(A well-established materials text with a design slant.)*



### Conversion of units – stress and pressure

<i>To →</i>	MPa	dyn/cm <sup>2</sup>	lb/in <sup>2</sup>	kgf/mm <sup>2</sup>	bar	long ton/in <sup>2</sup>
<i>From ↓</i>	<i>Multiply by</i>					
MPa	1	10 <sup>7</sup>	1.45 × 10 <sup>2</sup>	0.102	10	6.48 × 10 <sup>-2</sup>
dyn/cm <sup>2</sup>	10 <sup>-7</sup>	1	1.45 × 10 <sup>-5</sup>	1.02 × 10 <sup>-8</sup>	10 <sup>-6</sup>	6.48 × 10 <sup>-9</sup>
lb/in <sup>2</sup>	6.89 × 10 <sup>-3</sup>	6.89 × 10 <sup>4</sup>	1	7.03 × 10 <sup>-4</sup>	6.89 × 10 <sup>-2</sup>	4.46 × 10 <sup>-4</sup>
kgf/mm <sup>2</sup>	9.81	9.81 × 10 <sup>7</sup>	1.42 × 10 <sup>3</sup>	1	98.1	63.5 × 10 <sup>-2</sup>
bar	0.10	10 <sup>6</sup>	14.48	1.02 × 10 <sup>-2</sup>	1	6.48 × 10 <sup>-3</sup>
long ton/ in <sup>2</sup>	15.44	1.54 × 10 <sup>8</sup>	2.24 × 10 <sup>3</sup>	1.54	1.54 × 10 <sup>2</sup>	1

### Conversion of units – energy\*

<i>To →</i>	MJ	kWhr	kcal	Btu	ft lbf	toe
<i>From ↓</i>	<i>Multiply by</i>					
MJ	1	0.278	239	948	0.738 × 10 <sup>6</sup>	23.8 × 10 <sup>-6</sup>
kWhr	3.6	1	860	3.41 × 10 <sup>3</sup>	2.66 × 10 <sup>6</sup>	85.7 × 10 <sup>-6</sup>
kcal	4.18 × 10 <sup>-3</sup>	1.16 × 10 <sup>-3</sup>	1	3.97	3.09 × 10 <sup>3</sup>	99.5 × 10 <sup>-9</sup>
Btu	1.06 × 10 <sup>-3</sup>	0.293 × 10 <sup>-3</sup>	0.252	1	0.778 × 10 <sup>3</sup>	25.2 × 10 <sup>-9</sup>
ft lbf	1.36 × 10 <sup>-6</sup>	0.378 × 10 <sup>-6</sup>	0.324 × 10 <sup>-3</sup>	1.29 × 10 <sup>-3</sup>	1	32.4 × 10 <sup>-12</sup>
toe	41.9 × 10 <sup>3</sup>	11.6 × 10 <sup>3</sup>	10 × 10 <sup>6</sup>	39.7 × 10 <sup>6</sup>	30.8 × 10 <sup>9</sup>	1

MJ = megajoules; kWhr = kilowatt hour; kcal = kilocalorie; Btu = British thermal unit; ft lbf = foot-pound force; toe = tonnes oil equivalent.

### Conversion of units – power\*

<i>To →</i>	kW (kJ/s)	kcal/s	hp	ft lbf/s	Btu/h
<i>From ↓</i>	<i>Multiply by</i>				
kW (kJ/s)	1	4.18	1.34	735	4.47 × 10 <sup>4</sup>
kcal/s	0.239	1	0.321	176	1.07 × 10 <sup>4</sup>
hp	0.746	3.12	1	550	3.35 × 10 <sup>4</sup>
ft lbf/s	1.36 × 10 <sup>-3</sup>	5.68 × 10 <sup>-3</sup>	1.82 × 10 <sup>-3</sup>	1	
Btu/h	2.24 × 10 <sup>-5</sup>	9.33 × 10 <sup>-5</sup>	3.0 × 10 <sup>-5</sup>		1

kW = kilowatt; kcal/s = kilocalories per second; hp = horse power; ft lb/s = foot-pounds/second; Btu/h = British thermal units/hour