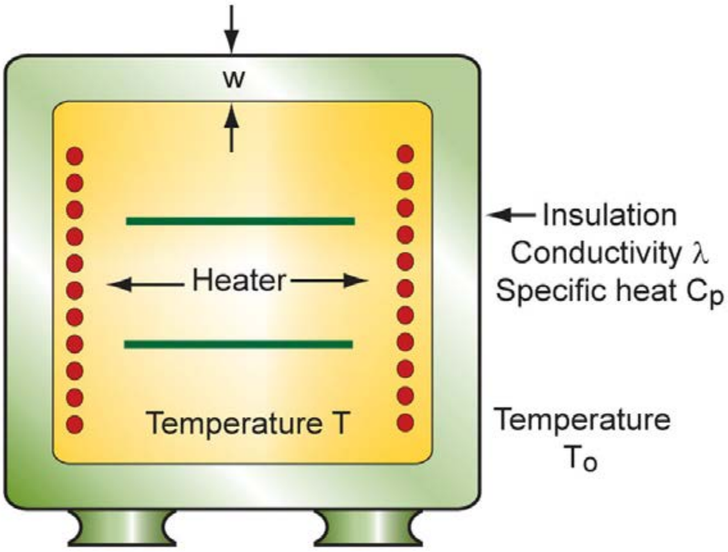
CES EduPack Case Studies:

Thermo-Mechanical Applications



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GRANTA TEACHING RESOURCES

This case study document is part of a set based on Mike Ashby's books to help introduce students to materials, processes and rational selection.
The Teaching Resources website aims to support teaching of materials-related courses in Design, Engineering and Science.
Resources come in various formats and are aimed primarily at undergraduate education.

About These Case Studies

These case studies were created with the help of Prof. Yves Brechet, Prof. David Embury, Dr. Norman Fleck, Dr. Jeff Wood, and Dr. Paul Weaver. Thanks also to Mr. Ken Wallace, the Director of the Cambridge University Engineering Design Centre and to the Engineering and Physical Sciences Research Council for their support of research into Materials Selection.

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Contents

1 Introduction

This document is a collection of case studies in Materials Selection. They illustrate the use of a selection methodology, and its software-implementation, the CES EduPack™. It is used to select candidate materials for a wide range of applications: mechanical, thermal, electrical, and combinations of these. Each case study addresses the question: out of all the materials available to the engineer, how can a short list of promising candidates be identified?

The analysis, throughout, is kept as simple as possible whilst still retaining the key physical aspects which identify the selection criteria. These criteria are then applied to materials selection charts created by CES EduPack, either singly, or in sequence, to isolate the subset of materials best suited for the application. Do not be put off by the simplifications in the analyses; the best choice of material is determined by function, objectives and constraints and is largely independent of the finer details of the design. Many of the case studies are generic: those for beams, springs, flywheels, pivots, flexible couplings, pressure vessels and precision instruments are examples. The criteria they yield are basic to the proper selection of a material for these applications.

There is no pretense that the case studies presented here are complete or exhaustive. They should be seen as an initial statement of a problem: how can you select the small subset of most promising candidates, from the vast menu of available materials? They are designed to illustrate the method, which can be adapted and extended as the user desires. Remember: design is open ended — there are many solutions. Each can be used as the starting point for a more detailed examination: it identifies the objectives and constraints associated with a given functional component; it gives the simplest level of modeling and analysis; and it illustrates how this can be used to make a selection. Any real design, of course, involves many more considerations. The 'Postscript' and 'Further Reading' sections of each case study give signposts for further information.

1.1 The Design Process

1. What are the steps in developing an original design?

   Answer

   - Identify market need, express as *design requirements*
   - Develop *concepts*: ideas for the ways in which the requirements might be met
   - *Embodiment*: a preliminary development of a concept to verify feasibility and show layout
   - *Detail design*: the layout is translated into detailed drawings (usually as computer files), stresses are analyzed and the design is optimized
   - *Prototyping*: a prototype is manufactured and tested to confirm viability

1.2 From Design Requirements to Constraints

2. Describe and illustrate the "Translation" step of the material selection strategy.

   Answer

   Translation is the conversion of design requirements for a component into a statement of function, constraints, objectives and free variables.

FUNCTION	What does the component do?
OBJECTIVE	What is to be maximized or minimized?
CONSTRAINTS	What non-negotiable conditions must be met?
FREE VARIABLE	What parameters of the problem is the designer free to change?

2 Energy-Efficient Kiln Walls

The energy cost of one firing-cycle of a large pottery kiln (Figure 2-1) is considerable. Part is the cost of the energy which is lost by conduction through the kiln walls; it is reduced by choosing a wall material with a low conductivity, and by making the wall thick. The rest is the cost of the energy used to raise the walls of the kiln and its contents to the operating temperature. It is reduced by choosing a wall material with a low heat capacity, and by making the wall thin. Is there a performance index which captures these apparently conflicting design goals? And if so, what is a good choice of material for kiln walls? The design requirements are listed in the Table 2-1.

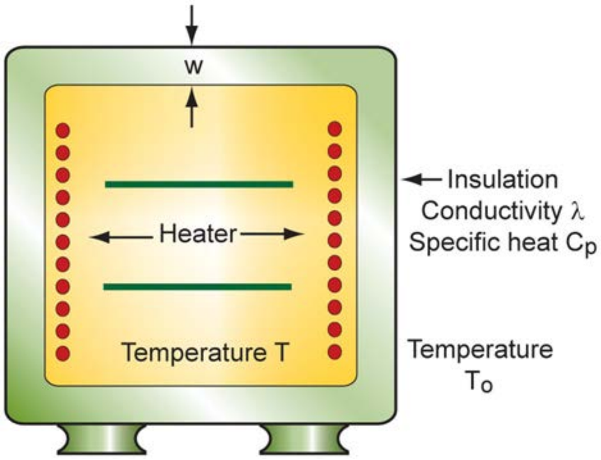


Figure 2-1. A kiln. In a firing cycle, energy is lost both by conduction and in heating the structure of the kiln itself.

Table 2-1. The design requirements

FUNCTION	Thermal insulation for kiln walls
OBJECTIVES	Minimize energy consumed in firing cycle Minimize capital cost of insulating material
CONSTRAINTS	Maximum operating temperature = 1000 K Possible limit on kiln wall-thickness, for space reasons

2.1 The Model

When a kiln is fired, the temperature rises quickly from ambient, T_o, to the firing temperature, T, where it is held for the firing time t (Figure 2-1). The energy consumed in the firing time has, as we have said, two contributions. The first is the heat conducted out. Once a steady-state has been reached, the heat loss per unit area by conduction, Q_1, is given by the first law of heat flow (Figure 2-1). Over the cycle time t (which we assume is long compared with the heat-up time) the heat loss is

$$Q_1 = \lambda \frac{dT}{dx} t = \lambda \frac{(T) - T_o}{w} \qquad \textit{(M 2.1)}$$

Here λ is the thermal conductivity, dT/dx is the temperature gradient and w is the wall thickness.

The second contribution is the heat absorbed by the kiln wall itself. Per unit area, this is

$$Q_2 = C_p \rho w \left(\frac{T - T_o}{2} \right) \qquad \textit{(M 2.2)}$$

where C_p is the specific heat and ρ is the density. The factor 2 enters because the average wall temperature is $(T - T_o)/2$. The total energy consumed per unit area of wall is the sum of these two heats:

$$Q = Q_1 + Q_2 = \frac{\lambda (T - T_o) t}{w} + \frac{C_p \rho w (T - T_o)}{2} \qquad \textit{(M 2.3)}$$

A wall which is too thin loses much energy by conduction, but absorbs little energy in heating the wall itself. One which is too thick does the opposite. There is an optimum thickness, which we find by differentiating equation (M 2.3) with respect to wall thickness w, giving:

$$w = \left(\frac{2\lambda t}{C_p \rho} \right)^{1/2} = (2\, at)^{1/2} \qquad \textit{(M 2.4)}$$

where $a = \lambda / C_p \rho$ is the thermal diffusivity. The quantity $(2at)^{1/2}$ has dimensions of length and is a measure of the distance heat can diffuse in time t . Equation (M 2.4) says that the most energy-efficient kiln wall is one that only starts to get really hot on the outside as the firing cycle approaches completion. That sounds as if it might lead to a very thick wall, so we must include a limit on wall thickness.

Substituting equation (M 2.4) back into equation (M 2.3) to eliminate w gives:

$$Q = (T - T_o)(2t)^{1/2} \left(\lambda\, C_p\, \rho \right)^{1/2}$$

Q is minimized by choosing a material with a low value of the quantity $\left(\lambda C_p r \right)^{1/2}$, that is, by maximizing

$$\boxed{M_1 = \left(\lambda\, C_p\, \rho \right)^{-1/2} = \frac{a^{1/2}}{\lambda}} \qquad \textit{(M 2.5)}$$

Now the limit on wall thickness. *A* given firing time, t , and wall thickness, w, defines, via equation (M 2.4), an upper limit for the thermal diffusivity, a:

$$a \leq \frac{w^2}{2t} \qquad \textit{(M 2.6)}$$

Selecting materials which maximize equation (M 2.5) with the constraint (M 2.6) minimizes the energy consumed per firing cycle.

Some candidates for the insulation could be very expensive. We therefore need a second index to optimize on cost. The cost of the insulation per unit area of wall is

$$C = C_m \, \rho \, w \qquad \textit{(M 2.7)}$$

where C_m is the material cost per kg. Substituting for w from equation (M 2.4) gives

$$C = (2t)^{1/2} \, C_m \left(\frac{\lambda \, \rho}{C_p}\right)^{1/2} = C_m \, \rho \, (2 \, at)^{1/2} \qquad \textit{(M 2.8)}$$

The cost of the material is minimized by maximizing

$$\boxed{M_2 = \frac{1}{C_m \rho}\left(\frac{1}{a}\right)^{1/2}} \qquad \textit{(M 2.9)}$$

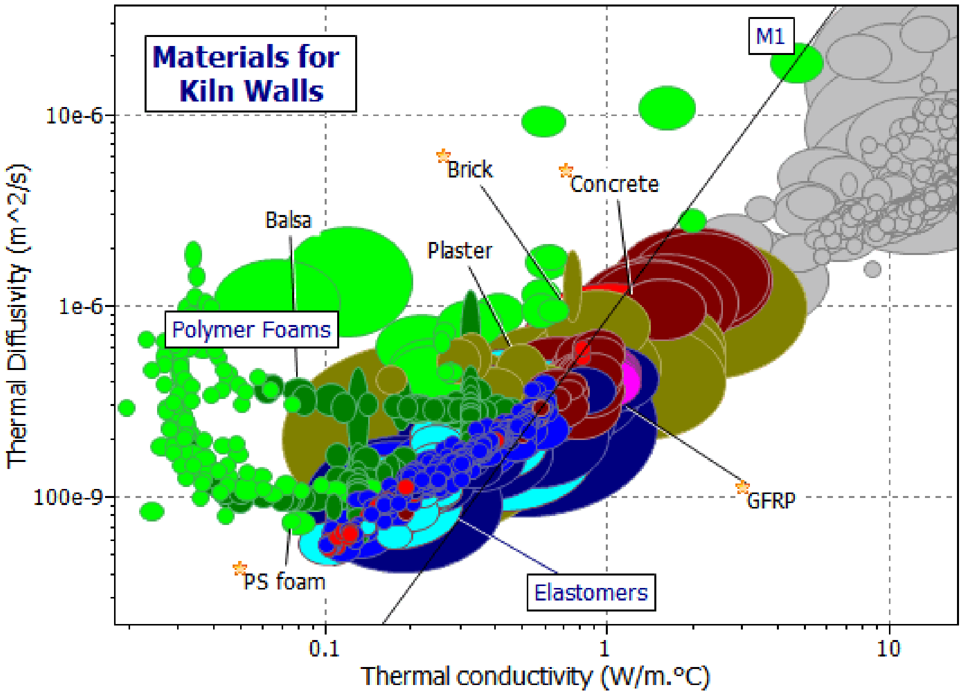
And, finally, the material must be able to tolerate an operating temperature of 1000 K.

2.2 The Selection

The neatest way to approach this problem is by a three-stage selection, starting with a chart of the thermal diffusivity (a compound-property)

$$a = \frac{\lambda}{C_p \, \rho}$$

plotted against thermal conductivity, λ , as in Figure 2-2. Contours of M_1 are lines of slope 2. One has been positioned at $M_1 = 10^{-3}$. To this can be added lines of constant wall thickness, corresponding to fixed values of the thermal diffusivity, *a* (equation (M 2.6)). The right-hand scale shows these limits, assuming a firing time of 6 hours; the horizontal broken line describes a thickness limit of 200 mm. We can now read-off the best materials for kiln walls to minimize energy, including the limit on wall thickness. Below the broken line, we seek materials which maximize M_1; while meeting the constraint on *w*.



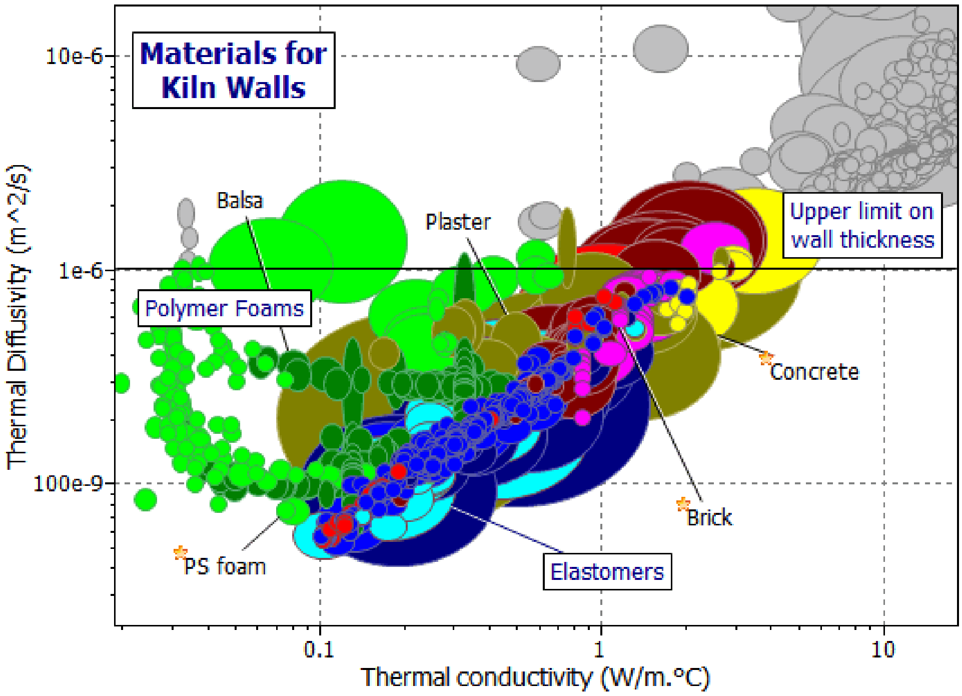


Figure 2-2. Thermal diffusivity, a , (a compound-property) plotted against thermal conductivity λ using the generic record subset. The selection line of slope 2 shows M_1; materials above the line are the best choice, provided they lie within the thickness-limit (right-hand scale and horizontal broken line).

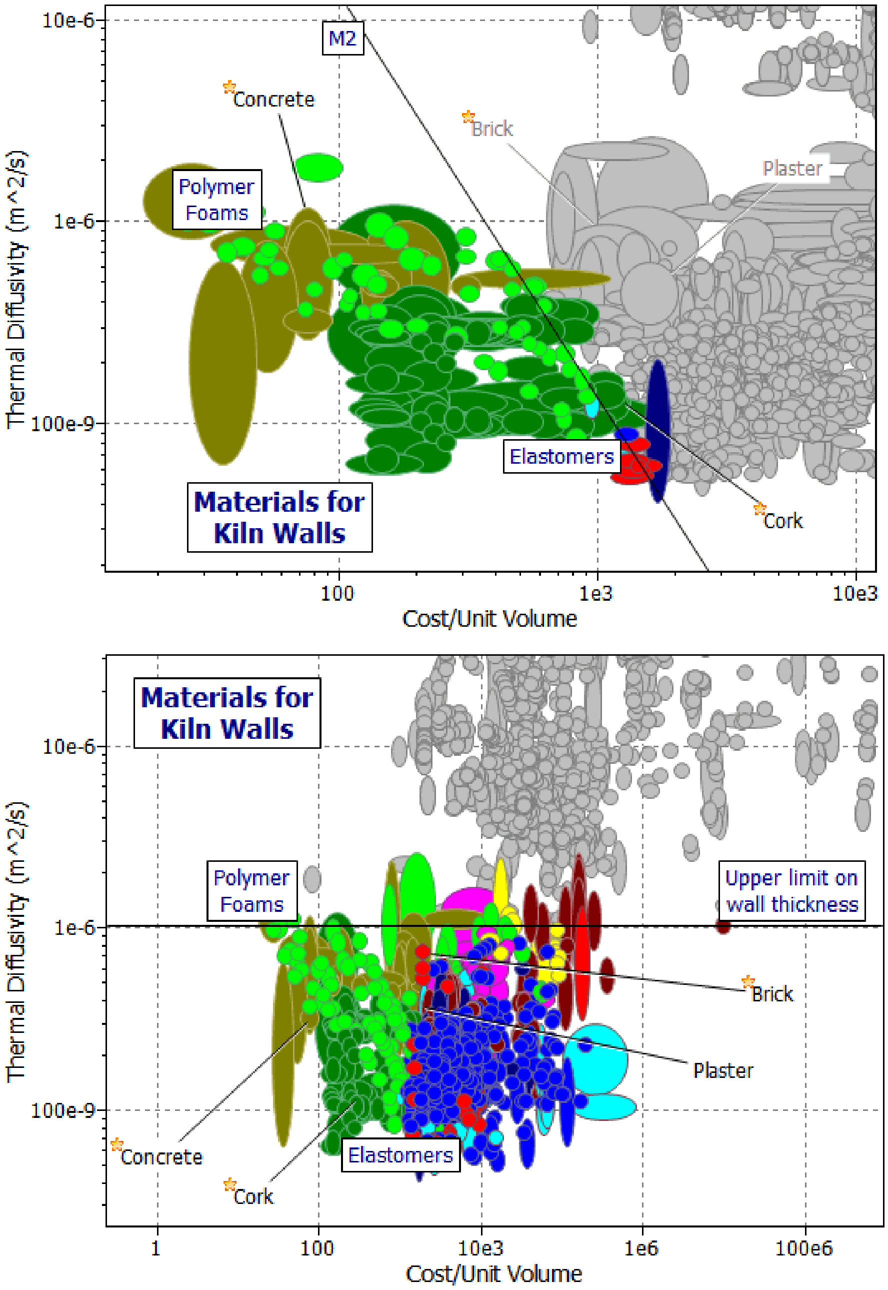


Figure 2-3. Thermal diffusivity, a , (a compound-property) plotted against cost per unit volume using the generic record subset. The selection line of slope -2 shows M_2 ; materials below the line are the best choice, provided they lie within the thickness-limit (right-hand scale and horizontal broken line).

The second stage optimizes a typical example of the cost of material for given firing conditions (equation (M 2.9)). The line shows M_2; once again limits on wall thickness can be added (right-hand scale and horizontal line).

The final stage is one for protection: it is a bar-chart of maximum operating temperature T_{max}. The line limits the selection to the region

$$T_{max} > 1000\ K$$

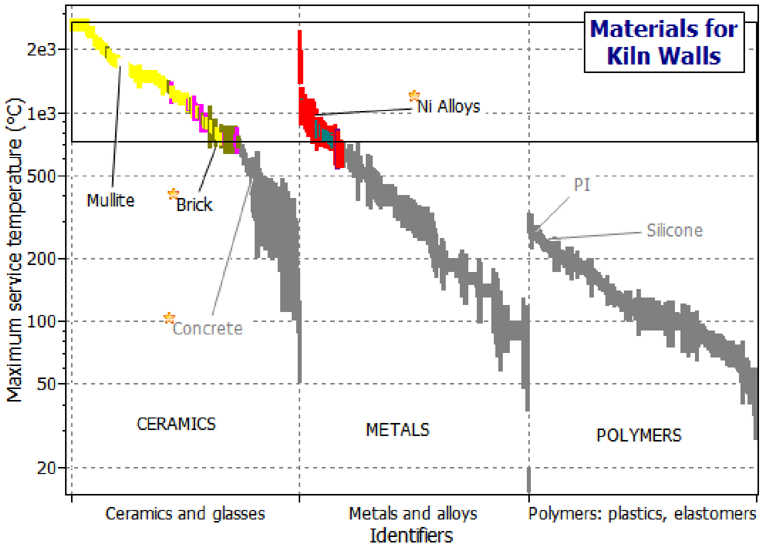


Figure 2-4. Maximum service temperature plotted against material class. Only metals and ceramics can tolerate temperatures as high as 1000 K; metals are eliminated by their high thermal conductivities.

Table 2-2 lists the results. Porous ceramics, including firebrick, are the obvious choice. But the degree of porosity is important. The more porous (low density) firebricks lie highest under the dashed line on Figure 2-2 — they require the thickest wall. So it may pay to use a denser firebrick, to meet the requirements on wall thickness.

Table 2-2. Materials for energy-efficient kilns

MATERIAL	$M_1 = a^{1/2}/\lambda$ ($m^2K/Ws^{1/2}$)	COMMENT
Porous Ceramics	$3 \times 10^{-4} - 3 \times 10^{-3}$	The obvious choice: the lower the density, the better the performance.
(Fiberglass)	10^{-2}	Thermal properties comparable with polymer foams; usable to 500°C.

2.3 Postscript

It is not generally appreciated that, in an efficiently-designed kiln, as much energy goes in heating up the kiln itself as is lost by thermal conduction to the outside environment. It is a mistake to make kiln walls too thick; a little is saved in reduced conduction-loss, but more is lost in the greater heat capacity of the kiln itself. That, too is the reason that foams are good: they have a low thermal conductivity *and* a low heat capacity. Centrally heated houses in which the heat is turned off at night suffer a cycle like that of the kiln. Here (because T_{max} is lower) the best choice is a polymeric foam, cork or fiberglass (which has thermal properties like those of foams). But as this case study shows — turning the heat off at night doesn't save you as much as you think, because you have to supply the heat capacity of the walls in the morning.

2.4 Further Reading

Holman, JP (1981) 'Heat Transfer' 5th Edition, McGraw-Hill, NY, USA.

3 Materials for Sauna Walls

If you build a sauna these days, you are concerned to minimize the energy it consumes. A sauna (Figure 3-1) like a kiln, has insulated walls to minimize the heat lost by conduction during the heating cycle. But if the heat capacity of the walls is high, a great deal of energy is lost simply in heating it up. So choosing the best material for a sauna wall requires a compromise between thermal conductivity λ and specific heat C_p . And it must also be cheap. Table 3-1 itemizes the design requirements.

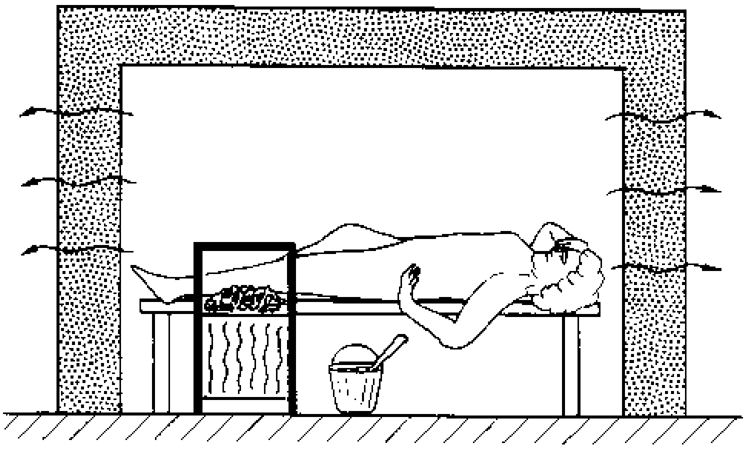


Figure 3-1. A Sauna. The material of the wall must insulate, at low heat capacity.

Table 3-1. The design requirements

FUNCTION	Thermal insulation for sauna walls
OBJECTIVE	Minimize energy consumed in use cycle
CONSTRAINTS	Maximum operating temperature = 90°C Low capital cost of insulation.

3.1 The Model

The Case Study "Energy-Efficient Kiln Walls" on page 3 analyzes the material requirements for thermal insulation chosen to minimize the total energy consumed during a heating cycle. The analysis for the sauna is the same as that for the kiln: we seek materials with high values of

$$M_1 = (\lambda C_p)\rho^{-1/2} = \frac{a^{1/2}}{\lambda} \qquad \text{(M 3.1)}$$

where λ is the thermal conductivity, (W/m.K), C_p the specific heat (J/kg.K), ρ the density (kg/m^3) and a the thermal diffusivity (m^2/s). The right thickness of a material with a large value of M_1 (Figure 3-2) minimizes the sum of the conduction losses and the energy used to heat the sauna walls themselves (which is lost when the sauna is switched off). The appropriate thickness, *w*, is given (see Case Study "Energy-Efficient Kiln Walls") by

$$w = \left(\frac{2\,\lambda\,t}{C_p \rho}\right)^{1/2} = (2\,at)^{1/2} \qquad \textit{(M 3.2)}$$

where t is the time for which the sauna is at its operating temperature (assumed to be long compared with the heat-up time). As with the kiln, we may wish to impose an upper limit on w for reasons of space. This implies an upper limit on diffusivity, a :

$$a \le \frac{w^2}{2t} \qquad \textit{(M 3.3)}$$

There is a second objective: than of minimizing material cost. The cost of the insulation is

$$C = C_m\,\rho\,w \qquad \textit{(M 3.4)}$$

per unit area of sauna wall. Substituting for w from equation (M 3.2) gives

$$C = (2t)^{1/2} C_m \left(\frac{\lambda\,\rho}{C_p}\right)^{1/2} \qquad \textit{(M 3.5)}$$

The cost is minimized by maximizing

$$\boxed{M_2 = \frac{1}{C_m}\left(\frac{C_p}{\lambda\,\rho}\right)^{1/2}} \qquad \textit{(M 3.6)}$$

3.2 The Selection

The neatest way to approach this problem, as with the Kiln of Case Study "Energy-Efficient Kiln Walls", is by a two-stage selection, starting with a chart of the thermal diffusivity (a compound-property)

$$a = \frac{\lambda}{C_p\,\rho}$$

plotted against thermal conductivity, λ , as in Figure 3-2. Contours of M_1 are lines of slope 2. One has been positioned at $M_1 = 10^{-3}$ ($m^2K/W.s^{1/2}$). To this can be added lines of constant wall thickness using equation (M 3.2); those shown assume a cycle time of 2 hours. Materials with high values of M_1 which also lie below the appropriate thickness contour minimize the total energy lost during the cycle.

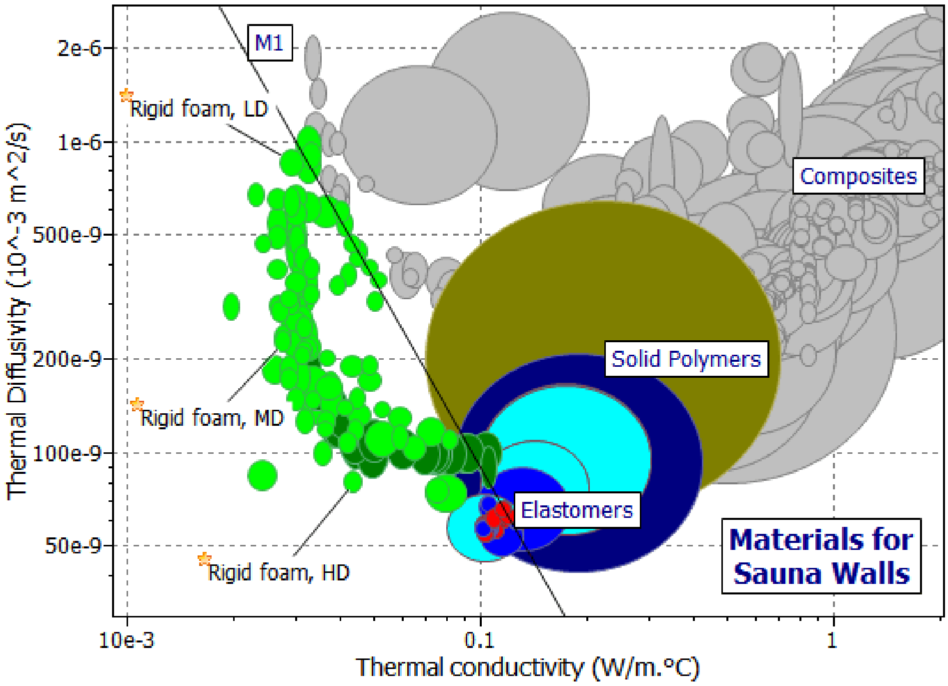


Figure 3-2. A chart of thermal diffusivity, a, plotted against thermal conductivity, λ. The line shows M_1.

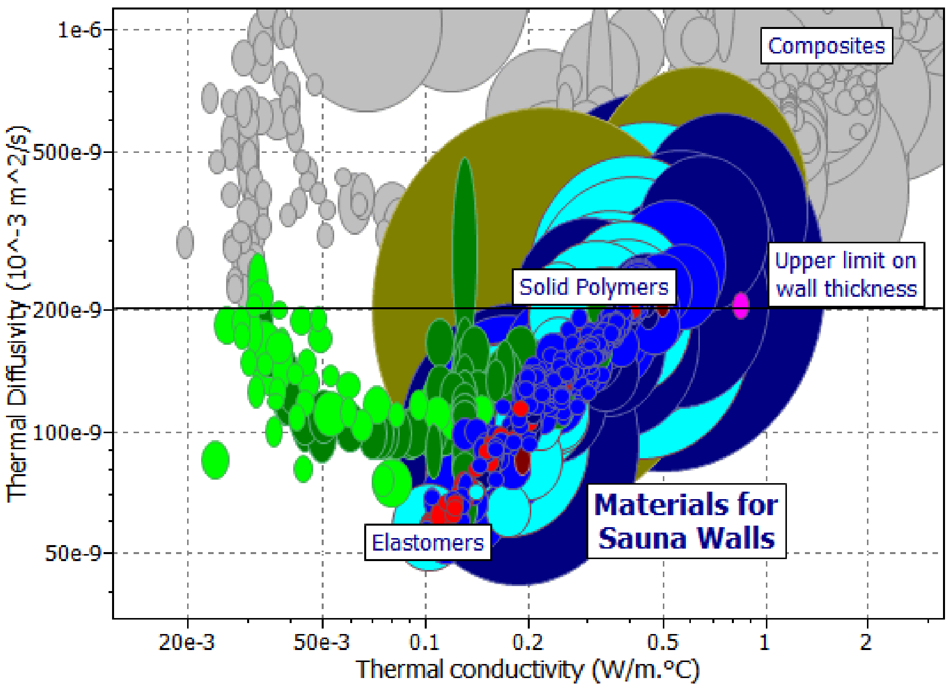


Figure 3-3. A chart of thermal diffusivity, a, plotted against thermal conductivity, λ. The line shows the wall thickness, w.

The second chart (Figure 3-3) allows selection to minimize cost, again allowing a constraint on wall thickness to be applied. The selection line for M_2 has slope -2. Materials which satisfy the conditions shown in the two charts are listed in Table 3-2.

3.3 Postscript

Traditionally, saunas were made of solid wood, many inches thick, built, often, like a log cabin. A wood-finished interior is part of the sauna culture, but (as the table shows) *solid* wood is not the best choice; its heat capacity is too high and its thermal conductivity — though low — is not as low as that of polymer foams or fiberglass. An energy-efficient sauna has an interior panelled in wood which is as thin as possible, consistent with sufficient mechanical strength; the real insulation, usually polymer foam or fiberglass, is invisible.

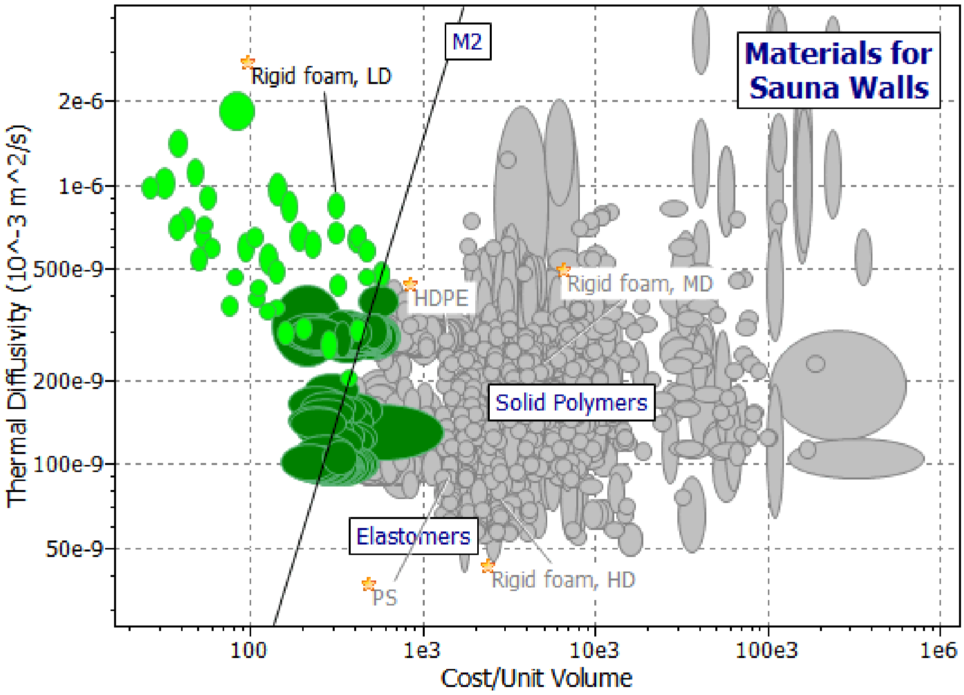


Figure 3-4. A chart of thermal diffusivity, a, plotted against cost per unit volume, $C_m\rho$. The line shows M_2.

Table 3-2. Materials for energy-efficient sauna walls

MATERIAL	$M_1 = a^{1/2}/\lambda$ (m2K/W.s$^{1/2}$)	COMMENT
Solid Elastomers	10-3 – 3 x 10-3	Good values of performance index. Useful if the wall must be very thin.
Solid Polymers	10-3 – 3 x 10-3	Limited to temperatures below 200°C.
Polymer Foam	3 x 10-3 – 3 x 10-2	The highest value of M1 — hence their use in house insulation. But limited to temperatures below 150°C
Woods	3 x 10-4 – 3 x 10-3	The boiler of Stevenson's 'Rocket' was insulated with wood.

3.4 Further Reading

Holman, JP (1981) '*Heat Transfer*', 5th Edition, McGraw-Hill, NY USA.

4 Minimizing Distortion in Precision Devices

The precision of a measuring device, like a sub-micrometer displacement gauge, is limited by its stiffness, and by the dimensional change caused by temperature gradients. Compensation for elastic deflection can be arranged; and corrections to cope with thermal expansion are possible too — provided the device is at a uniform temperature. Thermal gradients are the real problem: they cause a change of shape — that is, a distortion of the device, for which compensation is not possible. Sensitivity to vibration is also a problem: natural excitation introduces noise, and thus imprecision, into the measurement. So, in precision instrument design it is permissible to allow expansion, provided distortion does not occur (Chetwynd, 1987). Elastic deflection is allowed, provided natural vibration frequencies are high.

What, then, are good materials for precision devices? Table 4-1 lists the requirements.

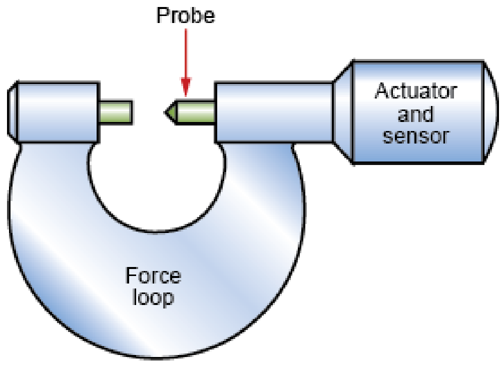


Figure 4-1. A precision device. All such devices have a force loop ; the precision depends on its dimensional stability.

Table 4-1. The design requirements

FUNCTION	Force loop (frame) for precision device
OBJECTIVE	Maximize positional accuracy (minimize distortion)
CONSTRAINTS	Must tolerate heat flux Must tolerate vibration Should not cost too much

4.1 The Model

Figure 4-1 shows, schematically, the features of such a device. It consists of a force loop, an actuator and a sensor. We aim to choose a material for the force loop. It will, in general, carry electrical components for actuation and sensing, and these generate heat. The heat flows into the force loop, setting up temperature gradients, and these in turn generate strain-gradients, or distortion. The relevant performance index is found by considering the simple case of one-dimensional heat flow through a beam with one surface exposed to a heat source (Figure 4-2).

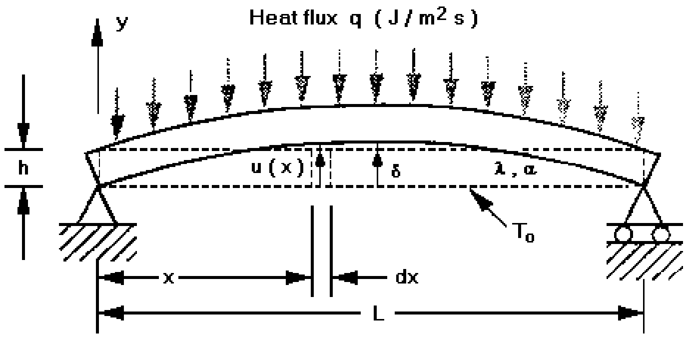


Figure 4-2. The distortion of a beam caused by a heat flux on one of its faces.

In the steady state, Fourier's law for one-dimensional steady-state heat flow states:

$$q = -\lambda \frac{dT}{dy} \qquad \text{(M 4.1)}$$

where q is heat input per unit area, λ is the thermal conductivity and *dT/dy* is the resulting temperature gradient. The thermal strain ε is related to temperature by

$$\varepsilon = \alpha(T_\circ - T) \qquad \text{(M 4.2)}$$

where α is the thermal expansion coefficient and T_o is ambient temperature.

A temperature gradient creates a strain gradient dε/dy in the beam, causing it, if unconstrained to take up a constant curvature K, such that:

$$K = \frac{d^2u}{dx^2} = \frac{d\varepsilon}{dy} = \alpha\frac{dT}{dy} = \frac{\alpha}{y}q \qquad \text{(M 4.3)}$$

where u is the transverse deflection of the beam. Integrating along the beam, accounting for the boundary conditions, gives an equation for the central deflection (distortion) δ:

$$\delta = C\,L^2 q\left(\frac{\alpha}{\lambda}\right) \qquad \text{(M 4.4)}$$

where C is a constant.

Thus for a given geometry and heat flux q, the distortion δ is minimized by selecting materials with large values of the index

$$M_1 = \frac{\lambda}{\alpha} \qquad \text{(M 4.5)}$$

The second problem is that of vibration. The sensitivity to external excitation is minimized by making the natural frequencies of the device as high as possible (Chetwynd, 1987; Cebon and Ashby, 1994). In general, it is the flexural vibrations which have the lowest frequencies; for a beam, their frequencies are proportional to

$$M_2 = \frac{E^{1/2}}{\rho} \qquad \text{(M 4.6)}$$

A high value of this index will minimize the problem.

4.2 The Selection

Here is an example in which the use of compound properties as axes is helpful. Figure 4-3 shows one way of tackling the problem, starting with the Generic record subset. The vertical axes shows the thermal-distortion index M_1 ; the horizontal axis is the stiffness index M_2.

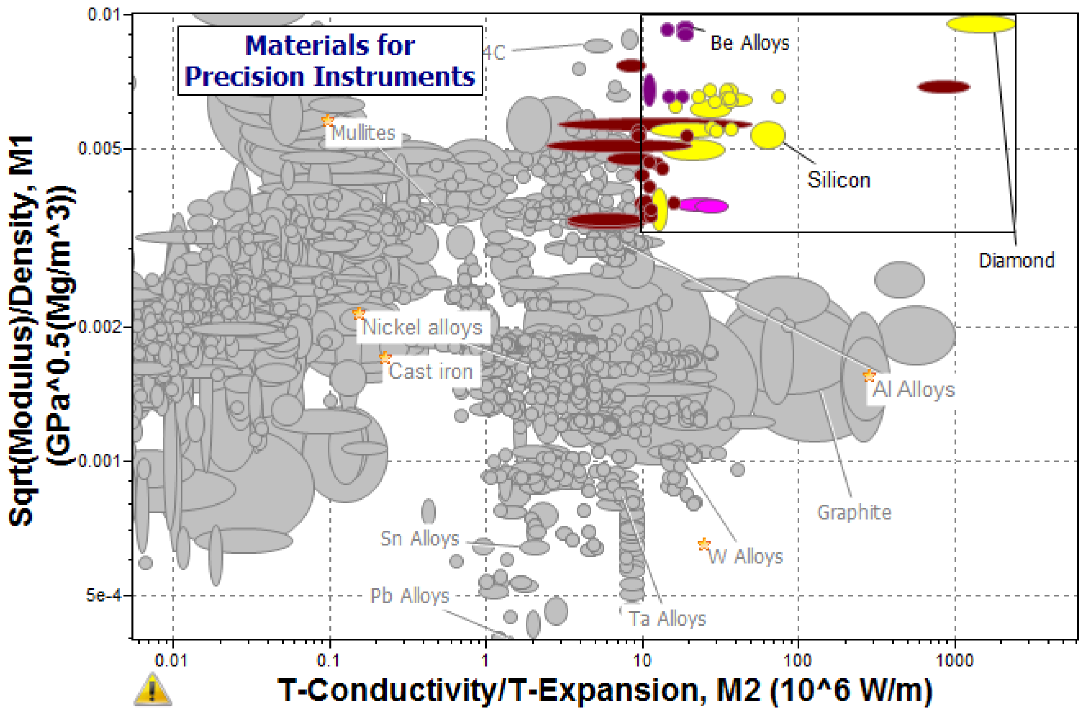


Figure 4-3. A chart of M_2 plotted against M_1

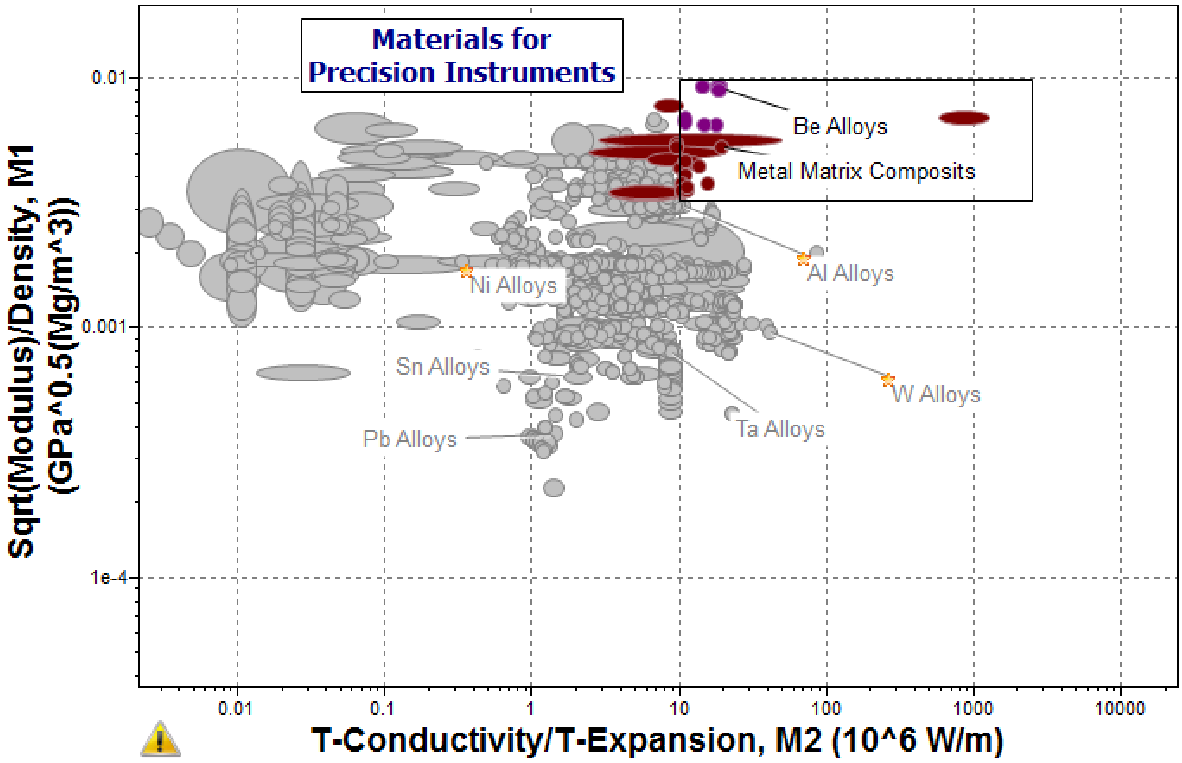


Figure 4-3. The same chart as Figure 4-2, but this time we concentrate on the light alloys branch of the materials tree

Figure 4-2 is a close-up view of the part of the chart which is interesting — the part with high M_1 and M_2. Steels, nickel and copper alloys are relatively poor by both criteria. The innovative choices lie at the top right. Diamond is outstanding, but practical only for the smallest devices (precision bearings, for example). Silicon carbide and aluminum nitride are excellent, but difficult to form to complex shapes. Silicon, an unexpected finding, is almost as good as the other fine ceramics and it *is* a practical choice: silicon is available cheaply, in large sections, and with high purity (and thus reproducibility). Silicon carbide is only slightly less good. The resulting short-list of candidates is provided in Table 4-2.

Light alloys feature in Table 4-2. It is worth examining them more closely. Figure 4-4 shows the results of plotting the light-alloy branch of the materials tree on the same axes. Among the light alloys, beryllium excels. But the Al-SiC metal-matrix composites are nearly as good; the composite Al-70% SiC(p) particularly so.

Table 4-2. Materials to minimize thermal distortion

MATERIAL	$M_1 = \lambda/a$ (W/m)	$M_2 = E^{1/2}/\rho$ ($GPa^{1/2}/(Mg/m^3)$)	COMMENT
Diamond	1.0 × 109	8.6	Outstanding M1 and M2; expensive.
Silicon	3 × 107	4.0	Excellent M1 and M2; cheap.
Aluminum nitride	3.5 × 107	5	Excellent M1 and M2; potentially cheap.
Silicon Carbide	4 × 107	6.2	Excellent M1 and M2; potentially cheap.
Beryllium	1.8 × 107	9	Outstanding M1; less good M2.
Metal matrix composites	up to 2 × 107	up to 6	A good choice
Aluminum alloys	107	2.6	Poor M1, but very cheap.
Tungsten	3 × 107	0.85	Better than copper, silver or gold, but less good than silicon, SiC, diamond
Molybdenum	2 × 107	1.3	
INVAR	3 × 107	1.4	

4.3 Postscript

Nano-scale measuring and imaging systems present the problem analyzed here. The atomic-force microscope and the scanning-tunnelling microscope both support a probe on a force loop, typically with a piezo-electric actuator and electronics to sense the proximity of the probe to the test surface. Closer to home, the mechanism of a video recorder and that of a hard disk drive qualify as precision instruments; both have an actuator moving a sensor (the read head) attached, with associated electronics, to a force loop. The materials identified in this case study are the best choice for force loop.

4.4 Further Reading

Chetwynd, DG (1987) Precision Engineering, 9, (1), 3.

Cebon, D and Ashby, MF (1994) Meas. Sci. and Technol., 5, 296.

5 Ceramic Valves for Taps

Few things are more irritating than a dripping tap. Taps drip because the rubber washer is worn, or the brass seat is pitted by corrosion, or both. Could an alternative choice of materials overcome the problem? Ceramics wear well, and they have excellent corrosion resistance in both pure and salt water. How about a tap with a ceramic valve and seat

Figure 5-1 shows a possible arrangement. Two identical ceramic discs are mounted one above the other, spring-loaded so that their faces, polished to a tolerance of 0.5 mm, are in contact. The outer face of each has a slot which registers it, and allows the upper disc to be rotated through 90° (1/4 turn). In the 'off' position the holes in the upper disc are blanked off by the solid part of the lower one; in the 'on' position the holes are aligned. Normal working loads should give negligible wear in the expected lifetime of the tap. Taps with vitreous alumina valves are now available. The manufacturers claim that they do not need any servicing and that neither sediment nor hard water can damage them.

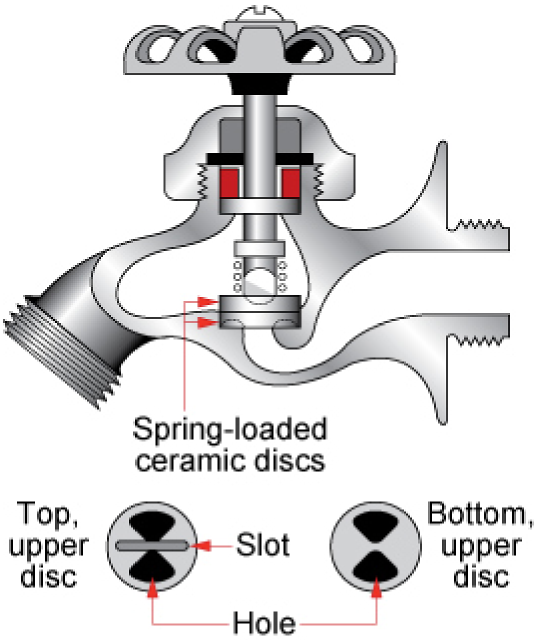


Figure 5-1. Taps: (a) The conventional tap as a valve and seat which wear, and are damaged by hard water. (b) Ceramic valves are resistant to wear and hard water.

But do they live up to expectation? As cold-water taps they perform well. But as hot-water taps, there is a problem: the discs sometimes crack. The cracking appears to be caused by thermal shock or by thermal mismatch between disc and tap body when the local temperature suddenly changes, as it does when the tap is turned on. Would another ceramic be better? The design requirements are summarized in Table 5-1.

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Table 5-1. The design requirements

FUNCTION	Ceramic valve
OBJECTIVE	Maximize life
CONSTRAINTS	Must withstand thermal shock High hardness to resist wear No corrosion in water

5.1 The Model

When the water flowing over the ceramic disc suddenly changes in temperature (as it does when you run the tap) the surface temperature of the disc changes suddenly by ΔT. The thermal strain of the surface is proportional to αΔT where α is the linear expansion coefficient; the constraint exerted by the interior of the disc generates a thermal stress

$$\sigma \approx E \propto \Delta T \qquad \textit{(M 5.1)}$$

If this exceeds the tensile strength of the ceramic, fracture will result. We require, for damage-free operation, that

$$\sigma \le \sigma_t \qquad \textit{(M 5.2)}$$

The safe temperature interval ΔT is therefore maximized by choosing materials with large values of

$$M_1 = \frac{\sigma_t}{E\,\alpha} \qquad \textit{(M 5.3)}$$

This self-induced stress is one possible origin for valve failures. Another is the expansion mismatch between the valve and the metal components with which it mates. The model for this is the almost the same; it is simply necessary to replace the thermal expansion coefficient of the ceramic, α, by the difference, Δα, between the ceramic and the metal.

5.2 The Selection

The thermal shock resistance of ceramics is summarized by Figure 5-2, which shows the index M_1. The horizontal line passes through alumina; promising candidates lie above the line. Table 5-2 summarizes the results.

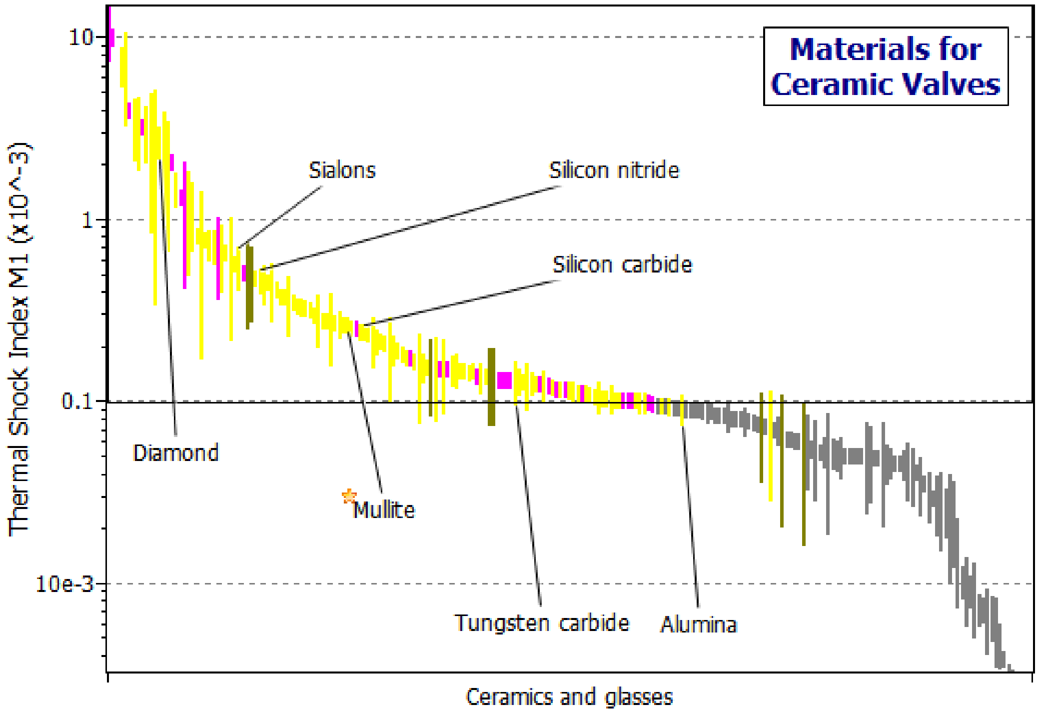

Figure 5-2. The index describing thermal shock resistance of ceramics. The horizontal line passes through alumina.

Table 5-2. Materials for ceramic valves

MATERIAL	COMMENT
Aluminas	Cheap, but poor thermal shock resistance
Silicon carbides	All are hard, corrosion resistant in water and most aqueous solutions, and have better thermal shock resistance than aluminas
Silicon nitrides	
Sialons	
Mullites	

5.3 Postscript

So ceramic valves for taps appear to be viable. The gain is in service life: the superior wear and corrosion resistance of the ceramic reduce both to a negligible level. But the use of ceramics and metals together raises problems of matching which requires careful redesign, and informed material selection procedures.

6 Materials for Storage Heaters

The demand for electricity is greater during the day than in the small hours of the night, for obvious reasons. It is not economic for electricity companies to reduce output, so they seek instead to smooth demand by charging less for off-peak electricity. Cheap, off-peak electrons can be exploited for home or office heating by using them to heat a large mass of thermal-storage material from which heat is later extracted when the demand — and cost — of power are at their peak.

The way such a storage heater works is shown schematically in Figure 6-1. A heating element heats a thermal mass during off-peak hours. During the expensive peak-demand hours the element is switched off and the thermostatically-controlled fan blows air over the hot mass, extracting heat and passing it to the room as needed.

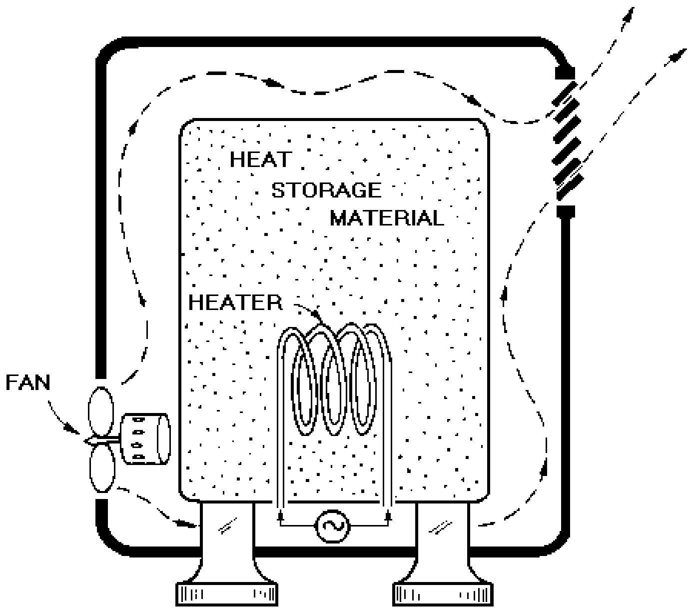

Figure 6-1. A storage heater. The heat-storage medium is chosen to have the highest heat capacity per unit cost.

What is the best material for the thermal mass? To hold enough heat to be useful, the thermal mass has to be large. It performs no other function, but just sits there, inert and invisible. No-one wants to pay more than they have to for inert, invisible mass. The best material is that which stores the most thermal energy (for a given temperature rise, ΔT) per unit cost. It must also be capable of withstanding indefinitely the temperature of the heater itself — that is, its maximum working temperature must exceed that of the surface temperature of the heating element. The design requirements are summarized in Table 6-1.

Table 6-1. The design requirements

FUNCTION	Heat storage
OBJECTIVE	Maximize heat stored per unit cost
CONSTRAINTS	Maximum service temperature > heater temperature

6.1 The Model

First, then, maximizing energy per unit cost. The thermal energy E stored in a mass m of solid when heated through a temperature interval ΔT is

$$E = m\, C_p\, \Delta T \qquad \text{(M 6.1)}$$

where C_p (kJ/kg.K) is the specific heat capacity of the solid (at constant pressure). The material cost is

$$C = m\, C_m \qquad \text{(M 6.2)}$$

where C_m is the cost per kg of the material. The energy stored per unit cost is therefore

$$\frac{E}{C} = \left(\frac{C_p}{C_m}\right) \Delta T \qquad \text{(M 6.3)}$$

This is the objective function. The energy per unit cost is maximized by maximizing

$$\boxed{M_1 = \frac{C_p}{C_m}} \qquad \text{(M 6.4)}$$

The constraint is that the maximum working temperature T_{max} be greater than the surface temperature of the heating element T_h, which we take to be 320°C, approximately 600 K.

$$T_{max} > T_h \qquad \text{(M 6.5)}$$

6.2 The Selection

Figure 6-2 shows the appropriate diagram: C_p/C_m plotted against T_{max}.

The selection results are listed in Table 6-2.

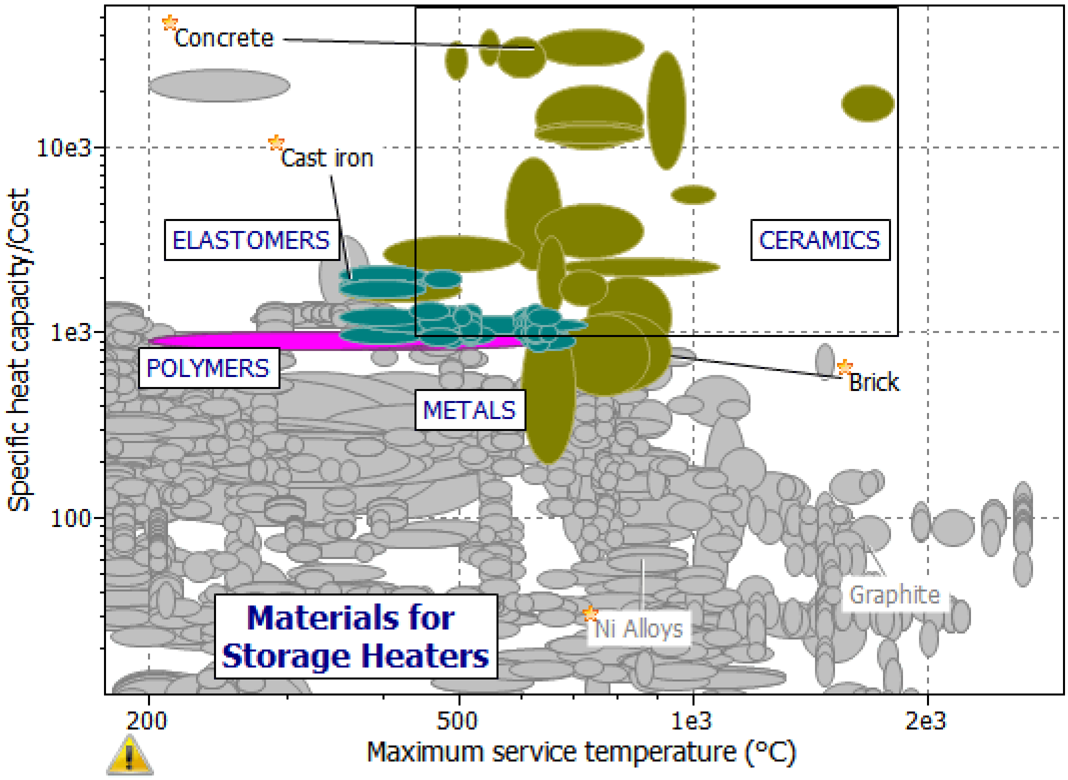

Figure 6-2. A chart showing heat capacity per unit cost plotted against maximum service temperature

Table 6-2. Materials for storage heaters

MATERIAL	C_p/C_m (MJ/£.K)	COMMENT
Stone (e.g. gravel)	1 – 30	A practical, cheap solution
Cement or concrete	10 – 20	Easy to shape, but maximum working temperature dangerously close to limit
Brick	2 – 9	Easy to assemble and disassemble, well suited for mass-produced product; high Tmax available.
Cast iron	1.5 – 2.5	Heavy, but otherwise a good choice.

6.3 Postscript

An important consideration is the rate at which heat can be extracted from the heater. This rate depends on the dimensions of the thermal mass and on the thermal diffusivity of the material of which it is made. Roughly speaking, the time-constant t for the cooling of a block of material of minimum dimension x is approximately

$$t = \frac{x^2}{2a} \qquad (M\ 6.6)$$

where the thermal diffusivity of the material is

$$a = \frac{\lambda}{C_p\, \rho} \qquad \textit{(M 6.7)}$$

λ is its thermal conductivity and ρ is its density. A large block cools slowly; small pieces cool more quickly if air can flow between them. So by breaking up the mass into loose gravel-like pieces, or by putting air channels through the brick, the rate of power output can be increased. In practice, the thermal diffusivity of the materials listed above (except for cast iron) all lie near 10^{-6} m^2/s. If the heat is to be extracted over a 6 hour period, then, according to equation (M 6.6) the block size should not exceed 0.2 m, otherwise the heat put in at night does not have time to leak out again during the day.

6.4 Further Reading

Holman, JP ‘Heat Transfer’, 5th Edition, (1981), McGraw-Hill, NY, USA.

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