Solution Manual: Materials: engineering, science, processing and design
2nd Edition

Part II: Chapters 8-13
Chapter 8  Fracture and fracture toughness

E8.1. What is meant by toughness? How does it differ from strength?

**Answer.** Strength is resistance to plastic flow and thus is related to the stress required to move dislocations through the solid. The initial strength is called the yield strength. Strength generally increases with plastic strain because of work hardening, reaching a maximum at the tensile strength.

Toughness is the resistance of a material to the propagation of a crack. A material with low fracture toughness, if it contains a crack, may fail before it yields. A tough material will yield, work harden even when cracked – the crack makes no significant difference.

E8.2. Why does a plastic zone form at the tip of crack when the cracked body is loaded in tension?

**Answer.** The intense stress field at the tip of a crack in a ductile material generates a process-zone: a region in which plastic flow takes place. The stress $\sigma_{\text{local}}$ rises as the crack tip is approached

$$\sigma_{\text{local}} = Y \frac{\sigma \sqrt{\pi c}}{2 \pi r}$$

As the crack tip is approached ($r$ is the radial distance from the crack tip, $c$ the crack length and $\sigma$ the remote stress). At the point where it reaches the yield strength $\sigma_y$, the material yields and – except for some work hardening – the stress cannot climb higher than this. The size of the plastic zone is found by setting $\sigma_{\text{local}} = \sigma_y$ and solving for $r$ giving

$$r_y = \left( \frac{\sigma^2 c}{2 \pi \sigma_y^2} \right) = \frac{K_I^2}{\pi \sigma_y^2}$$

(taking $Y = 1$). In reality the truncated part of the elastic stress field is re-distributed, making the plastic zone larger by a factor of 2.

E8.3. Why is there a transition from ductile to brittle behavior at a transition crack length, $c_{\text{crit}}$?

**Answer.** When cracks are small, materials yield before they fracture; when they are large, the opposite is true. When the crack is small, this stress is equal to the yield stress; when large, it falls off according to equation (taking $Y = 1$ again)

$$\sigma_f = \frac{K_{IC}}{\sqrt{\pi c}}$$

The transition from yield to fracture occurs when $\sigma_f = \sigma_y$, giving the transition crack length...
\[ c_{\text{crit}} = \frac{K_{\text{IC}}^2}{\pi \sigma_y^2} \]

**E8.4.** A tensile sample of width 10 mm contains an internal crack of length 0.3 mm. When loaded in tension the crack suddenly propagates when the stress reaches 450 MPa. What is the fracture toughness \( K_{\text{IC}} \) of the material of the sample? If the material has a modulus \( E \) of 200 GPa, what is its toughness \( G_c \)?

**Answer.** 9.77 MPa.m\(^{1/2}\)

**E8.5.** Use the \( K_{\text{IC}} - E \) chart of Figure 8.8 to establish

- Whether CFRP has higher fracture toughness \( K_{\text{IC}} \) than aluminum alloys?
- Whether polypropylene (PP) has a higher toughness \( G_c \) than aluminum alloys
- Whether polycarbonate (PC) has higher fracture toughness \( K_{\text{IC}} \) toughness than glass.

**Answer.**
- CFRP has lower fracture toughness \( K_{\text{IC}} \) than aluminum alloys.
- Polypropylene (PP) has a higher toughness \( G_c \) than aluminum alloys.
- Polycarbonate (PC) has a much higher fracture toughness \( K_{\text{IC}} \) than glass

**E8.6.** Find epoxy, soda glass and GFRP (epoxy reinforced with glass fibers) on the chart of Figure 8.8 and read off an approximate mean value for the toughness, \( G_c \) for each. Explain how it is that the toughness of the GFRP is so much larger than that of either of its components.

**Answer.** The approximate toughness \( G_c \) for the 3 materials are

- Epoxy 0.1 – 1 kJ/m\(^2\)
- Soda glass 0.05 – 0.07 kJ/m\(^2\)
- GFRP 3 – 20 kJ/m\(^2\)

CFRP is tougher than either of its components because of the energy dissipated in the process zone as broken fibers of glass are pulled out of the epoxy matrix.

**E8.7.** Use the chart of Figure 8.8 to compare the fracture toughness, \( K_{\text{IC}} \), of the two composites GFRP and CFRP. Do the same for their toughness, \( G_c \). What do the values suggest about applications they might best fill?

**Answer.** GFRP and CFRP have almost the same value-ranges of fracture toughness, \( K_{\text{IC}} \), but differ in their values of toughness, \( G_c \). GFRP, with the higher toughness, is the better choice for energy absorbing applications. CFRP, with higher stiffness and the same fracture toughness as GFRP, is the better choice for load-limited applications.

**E8.8.** Use the \( K_{\text{IC}} - \sigma_y \) chart of Figure 8.9 to find

- the range of transition crack sizes for stainless steel
- the range of transition crack sizes for polycarbonate (PC)
- the range of transition crack sizes for silicon nitride (Si\(_3\)N\(_4\)).

**Answer.** The approximate transition crack sizes \( c_{\text{crit}} \) for the 3 materials are

- Stainless steel 10 – 1000 mm
- Polycarbonate 5 – 12 mm
- Silicon nitride 0.02 – 0.03 mm
Exploring design with the CES EduPack software (Use Level 2 unless otherwise stated)

E8.9. Make a bar chart with \( \sigma_f = \frac{K_{IC}}{\sqrt{\pi c}} \) for an internal crack of length \( 2c = 1 \text{ mm} \) plotted on the y-axis. (use the Advanced facility to form the function)

Fracture toughness \( I(3.142 \times 0.0005)^{0.5} \).

Which materials have the highest values? Add an axis of density, \( \rho \). Use the new chart to find the two materials with highest values of \( \sigma_f / \rho \).

**Answer.** Low alloy steels and stainless steels.

![Fracture toughness vs Density graph](image)

E8.10. Suppose that the resolution limit of the NDT facility available to you is 1mm, meaning that it can detect cracks of this length or larger. You are asked to explore which materials will tolerate cracks equal to or smaller than this without brittle fracture. Make a bar chart with \( \sigma_f = \frac{K_{IC}}{\sqrt{\pi c}} \) for an internal crack of length \( 2c = 1 \text{ mm} \) plotted on the y-axis, as in the previous exercise. Add yield strength, \( \sigma_y \) on the x-axis. The material will fracture in tension if \( \sigma_f < \sigma_y \), and it will yield, despite being cracked, if \( \sigma_f > \sigma_y \). Plot either on a print out of the chart or in CES EduPack (using the line selection tool) a line of slope 1 along which \( \sigma_f = \sigma_y \). All the materials above the line will yield, all those below will fracture. Do age-hardening aluminum alloys lie above the line? Does CFRP?

**Answer.** Age-hardened aluminum alloys lie above the line, CFRP does not.

E8.11. Find data for PVC in Level 3 and make a plot like that of Figure 8.14 showing how fillers, blending and fibers influence modulus and toughness.

**Answer.**
Exploring the origins of surface energy, $\gamma$ J/m$^2$. The toughness, $G_c$, cannot be less than $2\gamma$ because two new surfaces are created when a material is fractured. What determines $\gamma$ and how big is it? The text explained how bonds are broken and atoms separated when new surface is created. Its was shown in Section 8.4 that

$$\gamma \approx \frac{1}{3}H_c r_o$$

Where $H_c$ is the cohesive energy in J/m$^3$ and $r_o$ is the atomic radius in m. To convert $H_c$ in kJ/mol (as it is in the database) into these units, multiply it by 106/ Molar volume and to convert $r_o$ from nm into m multiply by $10^{-9}$. All this can be done using the Advanced facility in the axis-choice dialog box.

Make a chart with $\gamma$ calculated in this way on the x-axis, and the measured value, “Surface energy, solid” on the y-axis and see how good the agreement is.

Answer.

![Graph showing the relationship between surface energy and other properties](image-url)
**E8.13.** If you pull on an atomic bond, it breaks completely at a strain of about 0.1. If the atom spacing is $a_0$, then breaking it requires a displacement $\delta = 0.1a_0$ and doing so creates two new surfaces, each of area $a_0^2$. If the bond stiffness is $S$, then the work done to create the new surfaces, per unit area, is

$$\gamma \approx \frac{1}{2a_0^2} \left( \frac{1}{2} S \delta \right) = \frac{Ea_0}{40} = \frac{1}{20} E r_0$$

(using equation 4.17 for $S$) where $r_0$ is the atomic radius. Use the CES EduPack Elements DB to explore whether real surface energies can be explained in this way. (Watch out for the units.)

**Answer.**

![Graph showing surface energies and atomic radii](image)

**E8.14.** Observe the general magnitudes of surface energies $\gamma$: they are about 1.5 J/m$^2$. Thus the minimum value of for $G_c$ should be about $2\gamma$ or 3 J/m$^2$. Return to the CES EduPack Level 3 database and find the material with the lowest value of $G_c$, which you can calculate as $K_{IC}^2 / E$. Is it comparable with $2\gamma$? Limit the selection to metals and alloys, polymers, technical ceramics and glasses only, using a Tree stage (materials such as foam have artificially low values of $G_c$ because they are mostly air).

**Answer.**

![Graph showing fracture toughness and Young's modulus](image)
Chapter 9  Shake, rattle and roll: cyclic loading, damage and failure

E9.1. What is meant by the Mechanical loss coefficient, $\eta$, of a material? Give examples of designs in which it would play a role as a design-limiting property.

**Answer.** The mechanical loss-coefficient or damping coefficient, $\eta$ (a dimensionless quantity), measures the degree to which a material dissipates vibration energy. If an elastic material is loaded, energy is stored. When it is unloaded some energy is returned, but not all. The difference is called the loss coefficient, $\eta$. It is the fraction of the stored elastic energy that is not recovered on unloading; instead it appears as heat. Applications in which resonance or fast elastic response is required (bells, high-speed relays and springs) require materials with low $\eta$. Applications in which it is desirable to damp vibration (sound isolation of buildings, suppression of vibration in machine tools) use material with high $\eta$.

E9.2. What is meant by the endurance limit, $\sigma_e$, of a material?

**Answer.** For many materials there exists a fatigue or endurance limit, $\sigma_e$ (units: MPa). It is the stress amplitude $\sigma_a$, about zero mean stress, below which fracture does not occur, or occurs only after a very large number ($N_f > 10^7$) cycles. Design against high-cycle fatigue is therefore very similar to strength-limited design, but with the maximum stresses limited by the endurance limit $\sigma_e$ rather than the yield stress $\sigma_y$.

E9.3. What is the Fatigue ratio? If the tensile strength $\sigma_{ts}$ of an alloy were 900 MPa, what, roughly, would you expect its endurance limit $\sigma_e$ to be?

**Answer.** The fatigue ratio $R_f$ is the ratio of the endurance limit to the tensile stress: $R_f = \frac{\sigma_e}{\sigma_{ts}}$

Form metals and polymers $R_f \approx 0.33$. Thus the expected value of the endurance limit for the alloy is about 300 MPa.

E9.4. The figure shows a $S-N$ curve for AISI 4340 steel, hardened to a tensile stress of 1800 MPa.

What is the endurance limit?

If cycled for 100 cycles at an amplitude of 1200 MPa and a zero mean stress, will it fail?

If cycled for 100,000 cycles at an amplitude of 900 MPa and zero mean stress, will it fail?

If cycled for 100,000 cycles at an amplitude of 800 MPa and a mean stress of 300 MPa, will it fail?

**Answer.**
- The endurance limit is about 630 MPa
- If cycled for 100 cycles at an amplitude of 1200 MPa and a zero mean stress
- If cycled for 100,000 cycles at an amplitude of 900 MPa and zero mean stress
- If cycled for 100,000 cycles at an amplitude of 800 MPa and a mean stress of 300 MPa
E9.5. The high-cycle fatigue life, \( N_f \), of an aluminum alloy is described by Basquin’s law:

\[
\frac{\Delta \sigma}{2} = \sigma_a = 480 \left( N_f \right)^{-0.12}
\]

(stress in MPa). How many cycles will the material tolerate at a stress amplitude \( \sigma_a \) of ±70 MPa?

**Answer.** \( 9.3 \times 10^6 \).

E9.6. The low-cycle fatigue of an aluminum alloy is described by Coffin’s law:

\[
\frac{\Delta \sigma}{2} = \sigma_a = 600 \left( N_f \right)^{-0.33}
\]

(stress in MPa). How many cycles will the material tolerate at a stress amplitude \( \sigma_a \) of ±100 MPa?

**Answer.** 216

E9.7. A material with a tensile stress \( \sigma_{ts} = 350 \) MPa is loaded cyclically about a mean stress of 70 MPa. If the stress range that will cause fatigue fracture in \( 10^5 \) cycles under zero mean stress is ±60 MPa, what stress range about the mean of 70 MPa will give the same life?

**Answer.** \( \frac{\Delta \sigma}{2} = 48 \) MPa.

E9.8. A component made of the AISI 4340 steel with a tensile strength of 1800 MPa and the S-N curve shown in Exercise E9.4 is loaded cyclically between 0 and 1200 MPa. What are the \( R \) value and the mean stress, \( \sigma_m \)? Use Goodman’s rule to find the equivalent stress amplitude for and \( R \)-value of −1, and read off the fatigue life from the S-N curve.

**Answer.** ±900 MPa. The life is \( 10^5 \) cycles.

E9.9. A material has a threshold cyclic stress intensity \( \Delta K_{th} \) of 2.5 MPa.m\(^{1/2}\). If it contains an internal crack of length 1 mm will it be safe (meaning, no failure) if subjected to continuous cyclic range of tensile stress \( \Delta \sigma \) of 50 MPa?

**Answer.** Yes. A stress range of \( \Delta \sigma = 62 \) MPa is needed to exceed the threshold.

**Exploring design with CES EduPack**

E9.10. Make a bar chart of mechanical loss coefficient, \( \eta \). Low loss materials are used for vibrating systems where damping is to be minimized – bells, high-frequency relays and resonant systems. High loss materials are used when damping is desired – sound deadening cladding for buildings, cars and machinery, for instance. Use the chart to find:

- the metal with the lowest loss coefficient.
- the metal with the highest loss coefficient.
Do their applications include one or more of those listed above?

**Answer.** The metal with the lowest loss coefficient is bronze. Its applications include bells. The metal with the highest loss coefficient is commercially pure lead. Its applications include roof and wall cladding.

**E9.11.** Use the Search facility to search materials that are used for

a) bells
b) cladding

**Answer.**

<table>
<thead>
<tr>
<th>Bells</th>
<th>Cladding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borosilicate glass</td>
<td>Glass ceramic</td>
</tr>
<tr>
<td>Soda-lime glass</td>
<td>Stone</td>
</tr>
<tr>
<td>Medium carbon steel</td>
<td>GFRP, epoxy matrix (isotropic)</td>
</tr>
<tr>
<td>Bronze</td>
<td>Lead alloys</td>
</tr>
<tr>
<td></td>
<td>Polyvinylchloride (tpPVC)</td>
</tr>
<tr>
<td></td>
<td>Granite</td>
</tr>
<tr>
<td></td>
<td>Bronze</td>
</tr>
<tr>
<td></td>
<td>Age-hardening wrought Al-alloys</td>
</tr>
<tr>
<td></td>
<td>Commercially pure zinc</td>
</tr>
<tr>
<td></td>
<td>Commercially pure lead</td>
</tr>
<tr>
<td></td>
<td>Lead alloys</td>
</tr>
</tbody>
</table>

**E9.12.** Use a Limit stage, applied to the Surface treatment datatable, to find surface treatment processes that enhance fatigue resistance. To do this, change the selection table Level 2 Surface treatment, open a Limit stage, scroll down to Function of treatment and click on Fatigue resistance – Apply. Copy and report the results.

Repeat, using the Level 3 Surface treatment datatable.

**Answer.**

**Results at Level 2:**
- Carburizing and carbonitriding
- Grinding and mechanical polishing
- Induction and flame hardening
- Laser surface hardening and melting
- Nitriding

**Results at Level 3:**
- Boriding
- Carburizing and Carbonitriding
- Chromizing and Plasma Chromizing
- Grinding
- Induction and flame hardening
- Ion implantation
- Laser surface hardening and melting
- Nitriding
- Shot peening and laser peening
E9.13. Explore the relationship between fatigue ratio and strength for a heat-treatable Low alloy steel AISI 4340. The endurance limit $\sigma_e$ is stored in the database under the heading “Fatigue strength at $10^7$ cycles.”

Plot the fatigue-yield ratio $\sigma_e/\sigma_y$ against the yield strength $\sigma_y$, and

Plot the fatigue-tensile stress ratio $\sigma_e/\sigma_{ts}$ against the tensile strength $\sigma_{ts}$.

Use Level 3 of the database, apply a Tree stage to isolate the folder for the Low alloy steel, AISI 4340, then make the two charts, hiding all the other materials. How do you explain the trends?

**Answer.**

There is a well-defined drop in the fatigue ratios as the strength increases. That based on yield shows the strongest dependence. The fatigue ratio based on tensile stress is more nearly constant, ranging from 0.4 for the low strength tempers to 0.35 for the high strength tempers. The stronger dependence on strength shown by the yield-based ratio is caused by the greater work hardening in the softer materials.
Chapter 10  Keeping it all together: fracture-limited design

E10.1. Supersonic wind tunnels store air under high pressure in cylindrical pressure vessels – the pressure, when released, produces hypersonic rates of flow. The pressure vessels are routinely proof-tested to ensure that they are safe. If such a cylinder, of diameter 400mm and wall thickness 20 mm, made of a steel with a fracture toughness of 42 MPa.m$^{1/2}$, survives a proof test to 40 MPa (400 atmospheres), what is the length of the largest crack it might contain?

**Answer.** $c = 3.5$ mm

E10.2. You are asked to select a polymer to make a flexible coupling. The polymer must have a modulus greater than 2 GPa. The objective is to maximize the available flexure without fracture. Use the chart of Figure 10.4 to identify two good choices to meet these requirements. Are there any metals that are as good?

**Answer.** PA (Polyamide, Nylon). No metals are better.

E10.3. Crash barriers like car fenders must absorb energy without total fracture. The most effective are those that deform plastically, absorbing energy in plastic work, but they are not reusable. Fenders that remain elastic spring back after impact. For practical reasons the material must have a modulus greater than 10 GPa. Use the chart of Figure 10.4 to find non-metallic materials for elastic fenders, assuming that the overriding consideration is that the displacement before fracture is as great as possible (the constraint on modulus ensures that it absorbs enough energy).

**Answer.** Best is GFRP, second best is CFRP

E10.4. Materials with high toughness $G_c$ generally have high modulus. Sometimes, however, the need is for high toughness with low modulus, so that the component has some flexibility. Use the chart of Figure 10.4 to find the material (from among those on the chart) that has a modulus less than 0.5 GPa and the highest toughness $G_c$. List applications of this material that you can think of that exploit this combination of properties.

**Answer.** Leather. Shoes, belts (including, until fairly recently, belts to transmit power).

E10.6. The Figure shows, at (a), a cylindrical tie rod with a diameter 20mm. The plan is to use it carry a cyclic load with a stress range ± 200 kN. The figure also shows the S-N curve of the material of which it is to be made. Will it survive for without failure for at least $10^5$ cycles?

**Answer.** The stress amplitude ± 637 MPa. The cylindrical rod will survive comfortably.

E10.7. The component of the previous exercise was made and tested. It failed in less than in less than $10^5$ cycles. A post-mortem revealed that it had fractured at the root of a threaded end, shown at (b) in the figure. The threads have a depth of 1.5mm and a root radius of 0.2 mm. Given this additional information, how many cycles would you expect it to survive?

**Answer.** The stress concentration factor caused by a notch (Chapter 7, Figure 7.7) is

$$K_{sc} = \frac{\sigma_{max}}{\sigma_{nom}} = 1 + \alpha \left( \frac{c}{\rho_{sc}} \right)^{1/2}$$

With $\alpha = 2$ for tensile loading. The threads therefore create a stress concentration factor of 6.5. The local stress amplitude at the root of a thread is more than ± 4000 MPa. The component will certainly fail.
E10.8. An adhesive has a toughness $G_c = 100 \text{ J/m}^2$ and a shear strength $\sigma_s = 0.1 \text{ MPa}$. What must the dimensions of the bonded area of a lap-joint be if it is to carry an in-plane tensile $F_t$ of 100 N but allow peeling at a force $F_p$ of 5 N?

Answer. 20 x 200 mm

E10.9. The figure shows a component to be made from the high-strength aerospace alloy Ti-6Al-4V. It will be loaded cyclically at $\pm$ 210 MPa. How long will it last?

Answer. The stress concentration factor caused by a notch (Chapter 7, Figure 7.7) is

$$K_{sc} = \frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}} = 1 + \alpha \left( \frac{c}{\rho_{sc}} \right)^{1/2}$$

With $\alpha = 2$ for tensile loading. The change of section of the sample introduces a stress concentration factor of 3.3. Thus the local stress range $\pm$ 695 MPa. It will last for about $10^5$ cycles.

Exploring design with CES EduPack (use Level 2 Materials for all selections)

E10.10. Use the Search facility in CES EduPack to find materials that are used for

- pressure vessels
- connecting rods
- rail track

(search using the singular – e.g. pressure vessel – since that will find the plural too)

Answer.

(a) Pressure vessels

- CFRP, epoxy matrix (isotropic)
- Low alloy steel
- Copper
- Age-hardening wrought Al-alloys
- Non age-hardening wrought Al-alloys
- Titanium alloys
(b) Connecting rods
- Low alloy steel
- Age-hardening wrought Al-alloys
- Titanium alloys
- Rail track
- High carbon steel
- Medium carbon steel

**E10.11.** You are asked, as in exercise E10.2, to select a polymer to make a flexible coupling. The polymer must have a modulus $E$ greater than 2 GPa. The objective is to maximize the available flexure without fracture, and that means materials with high $K_{IC}/E$. Use a Limit stage to impose the constraint on $E$, then make use a Graph stage to make a chart for $K_{IC}$ and $E$, put on an appropriate selection line and move it until only 3 materials remain in the Results window. What are they? Rank them by price.

**Answer.**
- Acrylonitrile butadiene styrene (ABS)
- Polycarbonate (PC)
- Polyurethane (tpPUR)

**E10.12.** You are asked to recommend materials that have yield strength above 500 MPa and perform best in a design based on the leak-before-break criterion. Construct an appropriate limit stage and chart, put on the necessary selection line and list the three materials you would recommend.

**Answer.** Tough steels and stainless steels are the best choice. The three that score the highest are
- Low alloy steel
- Stainless steel
- Nickel

**E10.13.** Repeat the previous selection, applying instead a yield-before-break selection criterion.

**Answer.** Yield before break favors materials that have lower strength, so that they yield more easily. The three that score the highest are
- Brass
- Nickel
- Stainless steel
E10.14. A material is sought for a high performance con-rod, requiring that the index as high a value of the index $\sigma_e / \rho$, where $\sigma_e$ is the endurance limit (the fatigue strength at $10^7$ cycles). It must enough toughness to tolerate stress concentrations, requiring that $K_{IC} > 15$ MPa.m

**Answer.**
- CFRP, epoxy matrix (isotropic)
- Titanium alloys
- Wrought magnesium alloys
Chapter 11  Rub, slither and seize: friction and wear

E11.1. Define the coefficient of friction. Explain why it is independent of the area of contact of the sliding surfaces.

Answer. The coefficient of friction $\mu$ is defined as:

$$\mu = \frac{F_s}{F_n}$$

Where $F_n$ is the normal load pressing the sliding surfaces together $F_s$ is the force opposing the sliding motion. Surfaces are never perfectly flat. Thus when two surfaces are placed in contact they touch only at points where asperities meet. The load $F_n$ is supported solely by the contacting asperities. The real contact area $A_r$ is only a tiny fraction of the apparent, nominal, area $A_n$. Even small loads cause large contact stresses, enough to cause plastic deformation. The contact points flatten, forming junctions with a total area $A_r$. The total load transmitted across the surface is

$$F_n = A_r \sigma_y$$

Where $\sigma_y$ is the yield strength. Thus the real contact area is

$$A_r = \frac{F_n}{\sigma_y}$$

– it depends on the normal load but not on the nominal area $A_n$.

E11.2. Woods – both resinous wood from the pine family and hardwoods like lignum vitae (it is so dense it sinks in water) – were, historically, used for bearings – even clocks had hardwood bearings. Why? Think of your own ideas, then research the Web.

Answer. The resin in the wood acts as a lubricant. It is compliant enough to conform to the counterface of the bearing and is able to trap grit particles, embedding them in the wood where they cease to be damaging.

From [http://www.woodex-meco.com/woodhome.htm](http://www.woodex-meco.com/woodhome.htm) wood is arguably the oldest bearing material on the planet, in continuous use since the invention of the wheel. Gladiators in ancient Rome drove chariots with wooden bearings; Marie Antoinette rode to her untimely end in a cart whose axles were born in wood. Bearings made of lignum vitae bore the rudder shafts of ships in the golden age of sail, and when the steamship rose to power, its propeller shaft spun in lig bearings, too. Wood continues to this day to be the most "shaft-kindly" bearing material available. Even the latest engineering plastics fail to protect metal shafts as well as wood.

E11.3. Give examples, based on your experience in sport (tennis, golf, swimming, skiing, rock climbing, hang-gliding…) of instances in which friction is wanted and when it is not.

Answer. Friction wanted: grips of racquets; soles of running and climbing shoes; climbing gloves; tires and brakes of cycles and cars.

Friction not wanted: the sliding surfaces of skis; the transmission and gearing of bicycles and cars; of brake cables and of archery bows.

E11.4. Now a more challenging one. Do the same based, again on your experience in sport, of instances in which wear is desirable and in which it is not.

Answer. Wear wanted: wear is used to condition the tires of racing cars, to abrade an inner tube before applying a puncture patch, in running-in or wear-in of components
Wear not wanted: brake pads, cycle tires, all mechanical components with sliding surfaces.

E11.5. What are the characteristics of materials that are a good choice for use as brake pads?

Answer. Brake pad material must apply friction without vibration, wear or fade. That is not easy when you reckon that, to stop a one-tonne car from 100 kph (60 mph) requires that you find somewhere to put about 0.4 MJ. That's enough (if you don't lose any) to heat a kg of steel to 800 °C. Brake materials must apply friction at temperatures that can approach 800 °C, and they must do so consistently and without the stick-slip that causes "brake squeal". That means materials that can tolerate heat, conduct it away to limit temperature rise, and lubricate, since it is this that quenches squeal.

Brake pad materials consist of a matrix, an abrasive additive to increase friction, a reinforcement to help conduct heat and a lubricant to suppress vibration. A typical pad material has a phenolic matrix with particles of silicates, silicon carbide (sandpaper grit) to control friction, and graphite or MoS₂ as a lubricant. Those on a military jet, a 747 or a F1 car are carbon or ceramic, required to tolerate the high temperatures that are generated at the sliding surfaces.

E11.6. A 30 mm diameter plane bearing of length 20 mm is to be made of a material with the \( P - v \) characteristics shown in Figure 11.11 (b). If the bearing load is 300 N and the maximum rotation rate is 500 rpm, is it safe to run it dry? If not, is boundary lubrication sufficient? (Remember that the bearing pressure is the load divided by the projected area normal to the load.)

Answer. The mean bearing pressure is the load divided by the projected area normal to the load, giving 0.5 MPa. The sliding velocity in m/s at the bearing surface is the rotation speed (in revolutions per second) times the circumference of the bearing, giving 0.78 m/s. Plotting these on Figure 11.11(b) shows that in is unsafe to run the bearing dry, but that boundary lubrication is enough to protect it.

E11.7. A bronze statue weighing 4 tonnes with a base of area 0.8 m² is placed on a granite museum floor. The yield strength of the bronze is 240 MPa. What is the true area of contact, \( A_r \), between the base and the floor?

Answer. 1.7 x 10⁻⁴ m² (167 square mm)

E11.8. The statue of the previous example was moved to a roof courtyard with a lead floor. The yield strength of the lead is 6 MPa. What now is the true area of contact?

Answer. 6.8 x 10⁻³ m² (6680 square mm)

E11.9. How would you measure the true area of contact \( A_r \) between two surfaces pressed into contact? Use your imagination to devise ways.

Answer.

- Coat one surface with a mono-molecular layer of a fluorescent dye which is transferred to the other surface at the contact points
- If the two materials that meet at the surface are metallic, the electrical resistance across the surface is a measure of the area of contact.
- The thermal conductivity across the surface is a measure of the area of contact.
**Exploring design with CES EduPack** (Use Level 2 unless otherwise stated).

**E11.10.** Use the “Search” facility to find materials that are used as bearings. (Search on the singular – bearing – since that picks up the plural as well.)

**Answer.**
- Silicon carbide
- Silicon nitride
- Low alloy steel

**E11.11.** Use the “Search” facility to find materials that are used as for brake pads. Do the same for brake discs.

**Answer.**
- Aluminum/Silicon carbide composite
- Cast iron, gray
- Cast iron, ductile (nodular)

**E11.12.** Use a “Limit” selection stage applied to the Surface Treatment data-table to find processes that enhance wear resistance. (To do this, click on SELECT in the main tool bar, then on ‘1. Selection Data’, just below it. Select Level 2 Surface treatment in the dialog box. Open a Limit stage, open Function of treatment, and click on Wear resistance.) Explore the record for laser-based methods. What are its typical uses?

**Answer.**
- Anodizing
- Carburizing and carbonitriding
- Electroless plating
- Electroplating
- Hot dip coating (Galvanizing)
- Induction and flame hardening
- Laser surface hardening and melting
- Metal flame spray
- Nitriding
- Texturing
- Vapor metallizing (PVD)
- Vitreous enameling

Laser surface hardening is used to create hard, wear-resistant facings on tools, rolls for rolling mills, tips of rocker arms and cylinder bores in automotive and marine engines

**E11.13.** Follow the same procedure as that of the previous example to search for processes used to control friction. Explore the record for grinding and mechanical polishing. What are grinding wheels made of?
Answer.

- Carburizing and carbonitriding
- Electroless plating
- Electroplating
- Etching
- Grinding and mechanical polishing
- Induction and flame hardening
- Laser surface hardening and melting
- Metal flame spray
- Nitriding
- Polymer powder coating
- Texturing
**Chapter 12  Agitated atoms: materials and heat**

**E12.1** Define specific heat. What are its units? How would you calculate the specific heat per unit volume from the specific heat per unit mass? If you wanted to select a material for a compact heat-storing device, which of the two would you use as a criterion of choice?

**Answer.** The energy to heat 1 kg of a material by 1 K is called the heat capacity or specific heat. The measurement is usually made at constant pressure (atmospheric pressure) and thus is given the symbol $C_p$. Its units are J/kg.K. When dealing with gases, it is more usual to measure the heat capacity at constant volume (symbol $C_v$) and for gases this differs from $C_p$. For solids the difference is small. The specific heat per unit volume is that per unit mass, $C_p$, multiplied by the density, $\rho$:

$$ (C_p)_{vol} = C_p \rho $$

The best choice for a heat storing device of minimum mass is the material with the greatest value of $C_p$; the best choice for one of minimum volume (thus compact) is that with the greatest value of $(C_p)_{vol}$ provided, in both cases, the material meets all the other constraints (e.g. an adequate maximum service temperature).

**E 12.2** What two metals would you choose from the $\lambda - a$ chart of Figure 12.5 to maximize the thermal displacement of a bi-metallic strip actuator? If the bimetallic strip has a thickness 2mm and an average thermal diffusivity $a$ of $5 \times 10^{-5}$ /C, how long will it take to respond when the temperature suddenly changes?

**Answer.** Materials that lie far as apart on the expansion coefficient axis as possible: copper (or brass) and steel – probably not lead because it is too soft. Response time $\approx 0.02$ sec.

**E12.3.** A new alloy has a density of 7380 kg/m$^3$. Its specific heat has not yet been measured. How would you make and approximate estimate of its value in the normal units of J/kg.K? What value would you then report?

**Answer.** Use $\rho C_p \approx 2 \times 10^6$ J/m$^3$.K for all solid materials. The result is $C_p \approx 271$ J/kg.K.

**E12.4.** The same new alloy of the last exercise has a melting point of 1540 K. Its thermal expansion coefficient has not yet been measured. How would you make and approximate estimate of its value? What value would you then report?

**Answer.** Use $a T_m \approx 0.02$. The result is $a \approx 1.3 \times 10^{-5}$ /C.

**E12.5.** Interior wall insulation should insulate well, meaning low thermal conductivity, $\lambda$, but require as little heat as possible to warm up when the central heating system is turned on (if the wall absorbs heat the room stays cold). Use the $\lambda - a$ chart of Figure 12.5 to find the materials that do this best. (The contours will help.)

**Answer.** Polymer foams have the lowest values of $\lambda$ and at the same time lime on the lowest contour of volumetric specific heat.

**E12.6.** You notice that the ceramic coffee mugs in the office get too hot to hold about 10 seconds after pouring in the hot coffee. The wall thickness of the cup is 2mm.

(a) What, approximately, is the thermal diffusivity of the ceramic of the mug?

(b) Given that the volume specific heat of solids, $\rho C_p$, is more or less constant at $2 \times 10^6$ J/m$^3$.K, what approximately is the thermal conductivity of the cup material?

(c) If the cup were made of a metal with a thermal diffusivity of $2 \times 10^{-5}$ m$^2$/s, how long could you hold it?
Answer.
(a) The diffusivity \( a \approx 5 \times 10^{-7} \text{ m}^2/\text{s} \).
(b) The conductivity \( \approx 1 \text{ W/m.K} \)
(c) A tenth of a second.

E12.7. A structural material is sought for a low-temperature device for use at –20 C that requires high strength but low thermal conductivity. Use the \( \lambda - \sigma_y \) chart of Figure 12.6 to suggest two promising candidates (reject ceramics on the grounds that they are too brittle for structural use in this application).
Answer. CFRP or GFRP.

E12.8. A material is needed for a small, super-efficient pressurized heat exchanger. The text explained that the index for this application is \( M = \lambda \sigma_y \). Plot contours of this index onto a copy of the \( \lambda - \sigma_y \) chart of Figure 12.6 and hence find the two classes of materials that maximizes \( M \).
Answer. Copper alloys and tungsten alloys.

Exploring design with CES EduPack (use Level 2, Materials, for all selections)

E12.9. Use the Search facility of CES EduPack to find materials for
a) thermal insulation
   - Alumina
   - Rigid Polymer Foam (HD)
   - Rigid Polymer Foam (LD)
   - Rigid Polymer Foam (MD)
   - Cork
   - Ionomer (I)
   - Polyetheretherketone (PEEK)
   - Polypropylene (PP)
   - Polystyrene (PS)
   - Polyurethane (tpPUR)
   - Zirconia
   - Ceramic foam
b) heat exchangers
   - Copper alloys
   - Titanium alloys
   - Bronze
   - Brass
• Copper
• Commercially pure titanium
• Titanium alloys
• Metal foam

E12.10. The analysis of storage heaters formulated the design constraints for the heat-storage material, which can be in the form of a particle-bed or a solid block with channels to pass the air to be heated. Use the selector to find the best materials. Here is a summary of the design requirements. List the top six candidates in ranked order.

<table>
<thead>
<tr>
<th>Function</th>
<th>• Storage heater</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>• Maximum service temperature &gt; 150C</td>
</tr>
<tr>
<td></td>
<td>• Non flammable (a Durability rating)</td>
</tr>
<tr>
<td>Objective</td>
<td>• Maximize specific heat/Material cost, ( C_p / C_m )</td>
</tr>
<tr>
<td>Free variable</td>
<td>• Choice of material</td>
</tr>
</tbody>
</table>

Answer. Method: use a Limit stage to apply all the constraints; then make a Graph stage with Price \( C_m \) on the x-axis and Specific heat \( C_p \) on the y-axis. A line of slope +1 corresponds to the condition \( C_p / C_m \) = constant. Results: Concrete, cement, sandstone, marble, limestone, brick

E12.11. The requirements for a material for an automobile radiator, described in the text, are summarized in the table. Use CES EduPack to find appropriate materials to make them. List the top four candidates in ranked order.

<table>
<thead>
<tr>
<th>Function</th>
<th>• Automobile heat exchanger</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>• Elongation &gt; 10%</td>
</tr>
<tr>
<td></td>
<td>• Maximum use temperature &gt; 180C</td>
</tr>
<tr>
<td></td>
<td>• Price &lt; $5/kg</td>
</tr>
<tr>
<td></td>
<td>• Durability in fresh water = very good</td>
</tr>
<tr>
<td>Objective</td>
<td>• Maximize Thermal conductivity x Yield strength, ( \lambda \sigma_y )</td>
</tr>
<tr>
<td>Free variable</td>
<td>• Choice of material</td>
</tr>
</tbody>
</table>

Answer. Method: use a Limit stage to apply all the constraints; then make a Graph stage with Thermal conductivity \( \lambda \) on one axis and Yield strength \( \sigma_y \) on the other. A line of slope −1 corresponds to the condition \( \lambda \sigma_y \) = constant. Results: Age hardened wrought aluminum alloys, brass, copper, non age-hardened aluminium alloys.

E12.12. A structural material is sought for a low-temperature device for use at −20 C that requires high tensile strength \( \sigma_{ts} \) but low thermal conductivity \( \lambda \). For reasons of damage tolerance the fracture toughness \( K_{lc} \) must be greater than 15 MPa.m\(^{1/2}\). Apply the constraint on \( K_{lc} \) using a Limit stage, then make a chart with \( \sigma_{ts} \) on the x-axis and \( \lambda \) on the y-axis and observe which materials best meet the requirements.

Answer. • CFRP, epoxy matrix (isotropic)
• GFRP, epoxy matrix (isotropic)
E12.13 Interior wall insulation should insulate well, meaning low thermal conductivity, $\lambda$, but require as little heat as possible to warm up to the desired room temperature when the central heating system is turned on – that means low specific heat. The thickness of the insulation is almost always limited by the cavity space between the inner and outer wall, so we need materials that it is the specific heat per unit volume, not per unit mass that is important here. To be viable the material must have enough stiffness and strength to support its own weight and be easy to install – take that to mean a modulus $E > 0.01$ GPa and a strength $\sigma_y > 0.2$ MPa. Make a selection based on this information. List the materials you find that best meet the design requirements.

**Answer.**
- Cork
- Rigid Polymer Foam (HD)
- Rigid Polymer Foam (LD)
- Rigid Polymer Foam (MD)

E12.14 Here is the gist of an e-mail one of the authors received as this chapter was being written. “We manufacture wood stoves and fire places that are distributed all over Europe. We want to select the best materials for our various products. The important characteristics for us are: specific heat, thermal conductivity, density…and price, of course. What can your CES EduPack software suggest?”

Form your judgment about why these properties matter to them. Rank them, deciding which you would see as constraints and which as the objective. Consider whether there are perhaps other properties they have neglected. Then – given your starting assumptions of just how these stoves are used – use CES EduPack to make a selection. Justify your choice.

There is no “right” answer to this question – it depends on the assumptions you make. The essence is in the last sentence of the last paragraph: justify your choice.

**Example of an answer.** First, a quick think about the constraints: what’s missing? The most obvious is a lower limit on Maximum service temperature: if the material of the stove cannot tolerate the temperature at which it will be used, it is worthless. Second, the material will have to be shaped. Most wood-burning stoves have quite an intricate shape requiring casting, so we should experiment with the effect of a constraints requiring that the material can be cast. With this in mind, we can start. The e-mail gives no numeric limits on properties (needed for a Limit stage) so it is best to work with Graph stages.

1) The constraint on the ability to shape by casting seems likely to be important: it is applied with a Tree stage, selecting Casting from the ProcessUniverse.

2) I made the assumption that low material Price and adequate Maximum service temperature were important. The first figure shows these two properties with a red box surrounding those with high Max service temperature and low Price.

3) It seems probable that the makers want high specific heat so that the stove stores heat and high thermal conductivity to spread it. The two are plotted in the second figure. Stainless steels and nickel alloys do not do well; cast irons are better. Aluminum alloys are best by these criteria, but their service temperature looks like a problem.
4) Finally, density. It is plotted in the third figure. The lightest choices are aluminum and magnesium alloys. Cast irons and steels meet all the other requirements well, but are relatively dense.

The conclusions. This brief survey suggests that cast irons or cast steel are the most straightforward choice if casting is the preferred shaping method. The three figures provide a starting point for more detailed discussions with the maker, establishing answers to the unanswered questions, and eliciting a more precise statement of the relative priorities the maker attaches to each property. The exercise can be repeated quickly with the constraint on casting removed and replaces by one on shaping by other methods, or none at all. Taken together this forms the starting point for an effective consultation.

Figures 1 and 2. The first two charts. A Tree stage selecting materials that can be cast has already been applied. The red boxes suggest where selection criteria might be placed. CES '07 Level 2.

Figure 3. The densities of the materials that can be cast (CES '07 Level 2)
Exploring the science with CES EduPack Elements

**E12.15.** The text says that materials expand about 2% between 0K and their melting point. Use CES EduPack Elements to explore the truth of this by making a bar chart of the expansion at the melting point, $\alpha T_m/10^6$ (the $10^6$ is to correct for the units of $\alpha$ used in the database).

**Answer.** The figure shows that the great majority of elements have values of expansion between 0K and the melting point of between 1% and 3%.

![Thermal expansion coefficient at 300K vs. Melting temperature graph]

**E 12.16.** When a solid vaporizes, the bonds between its atoms are broken. You might then expect that latent heat of vaporization, $L_v$, should be nearly the same as the cohesive energy $H_c$, since it is basic measure of the strength of the bonding. Plot one against the other, using CES EduPack Elements. How close are they?

**Answer.** The two areas almost identical – the heat of vaporization is slightly less than the cohesive energy.

![Heat of Vaporization vs. Cohesive Energy graph]
E 12.17. When a solid melts, some of the bonds between its atoms are broken but not all – liquids still have a bulk modulus, for example. You might then expect that the latent heat of melting, $L_m$, should be less than the cohesive energy $H_c$, since it is basic measure of the strength of the bonding. Plot one against the other, using CES EduPack Elements ($L_m$ is called the Heat of fusion in the database). By what factor is $L_m$ less than $H_c$? What does this tell you about cohesion in the liquid?

**Answer.** The latent heat of melting is about 1/25th of the cohesive energy. Bonding in the liquid is nearly as strong as in the solid.

![Graph showing the relationship between cohesive energy and Heat of fusion](image)

E12.18. The latent heat of melting (Heat of fusion), $H_m$, of a material is said to be about equal to the heat required to heat it from absolute zero to its melting point, $C_p T_m$, where $C_p$ is the specific heat and $T_m$ is the absolute melting point. Make a chart with $H_m$ on one axis and $C_p T_m$ on the other. To make the comparison right we have to change the units of $H_m$ in making the chart to J/kg instead of kJ/mol. To do this multiply $H_m$ by

$$\frac{10^6}{\text{Atomic weight in kg / kmol}}$$

Using the “Advanced” facility in CES EduPack Elements. Is the statement true?

**Answer.** The chart shows that $H_m$ and $C_p T_m$ are strongly correlated, but the claim that they are equal is not accurate. A more accurate statement is that $H_m \approx 0.5 C_p T_m$.
E12.19. The claim was made in the text that the modulus $E$ is roughly proportional to the absolute melting point $T_m$. If you CES EduPack Elements to explore this correlation you will find that it is, in fact, very good (try it). That is because $T_m$ and $E$ are measured in different units and, from a physical point of view, the comparison is meaningless. To make a proper comparison, we use instead $k T_m$ and $E \Omega$ where $k$ is Boltzmann’s $(1.38 \times 10^{-23} \text{ J/K})$ constant and $\Omega \text{ m}^3/\text{atom}$ is the atomic volume. These two quantities are both energies, the first proportional to the thermal energy per atom at the melting point and the second proportional to the work to elastically stretch an atomic bond. It makes better sense, from a physical standpoint, to compare these.

Make a chart for the elements with $k T_m$ on the x-axis and $E \Omega$ on the y-axis to explore how good this correlation is. Correlations like these (if good) that apply right across the periodic table provide powerful tools for checking data, and for predicting one property—say, $E$—if the other—here $T_m$ is known. Formulate an equation relating the two energies that could be used for these purposes.

**Answer.** The figure shows the two quantities ($E$ has been multiplied by $10^9$, converting it to N/m$^2$ to make the units consistent). There is clearly a correlation between the two quantities. The line has slope 1. Its equation is

$$E \Omega = 90 k T_m$$

E12.20. The specific heat, once above the Debye temperature, is predicted to be $3 R$ where $R$ in units of kJ/per kmol.K. Make a plot for the elements with Debye temperature on the x-axis and specific heat in these units on the y-axis to explore this. You need to insert a conversion factor because of the units. Here it is, expressed in the units contained in the database

Specific heat in kJ/kmol.K = specific heat in J/kg.K x atomic weight in kg/kmol /1000.

Form this quantity dividing the result by $R = 8.314 \text{ kJ/kmol.K}$, and plot it against the Debye temperature. The result should be 3 except for materials with high Debye temperatures. Is it? Fit a curve by eye to the data. At roughly what temperature does the drop of first begin?
Answer. The figure shows that the specific heat, in kJ/ per kmol K, is close to $3R$ for elements with a Debye temperature below about 400 K. Those with larger Debye temperatures have lower specific heats because some of the vibration modes are not excited at room temperature (the temperature at which the specific heat is measured).

E12.21 Explore the mean free path of phonons $\ell_m$ in the elements using equation 12.10 of the text. Inverting it gives

$$\ell_m = \frac{\lambda}{\rho C_p c_o}$$

in which the speed of sound $c_o = \sqrt{E/\rho}$.

Use the Advanced facility in the CES EduPack Elements software to make a bar chart of $\ell_m$ for the elements. Which materials have the longest values? Which the shortest?

Answer. The figure shows $\ell_m$ in meters. (Young’s modulus has been multiplied by $10^9$, converting it to N/m$^2$, to make the units consistent. The shortest mean free path is that in sulfur—it is only slightly larger than the atom size. The longest is potassium. It is about 500 atom diameters.
Chapter 13  Running hot: using materials at high temperatures

E13.1. The self-diffusion constants for aluminum are $D_o = 1.7 \times 10^{-4}$ m$^2$/s and $Q_d = 142$ kJ/mol. What is the diffusion rate in aluminum at 400°C?

**Answer.** The diffusion rate is $D = 1.6 \times 10^{-15}$ m$^2$/s.

E13.2. A steel component is nickel plated to give corrosion protection. To increase the strength of the bond between the steel and the nickel, the component is heated for 4 hours at 1000°C. If the diffusion parameters for nickel in iron are $D_o = 1.9 \times 10^{-4}$ m$^2$/s and $Q_d = 284$ kJ/mol, how far would you expect the nickel to diffuse into the steel in this time?

**Answer.** $D = 3.77 \times 10^{-16}$ m$^2$/s; the approximate depth of diffusion is 2.3 microns.

E13.3. The diffusion coefficient at the melting point for materials is approximately constant, with the value $D = 10^{-12}$ m$^2$/s. What is the diffusion distance if a material is held for 12 hours at just below its melting temperature? This distance gives an idea of the maximum distance over which concentration gradients can be smoothed by diffusion.

**Answer.** The distance is 0.2 mm.

E13.4. What are the requirements of a creep-resistant material? What materials would you consider for use at 550°C?

**Answer.** The materials that best resist power law creep are those with high melting points, since diffusion and thus creep-rates scale as $T / T_m$, and a microstructure that maximizes obstruction to dislocation motion through alloying to give a solid solution and precipitate particles. The materials that best resist diffusional flow are those with a high melting temperature and a large grain size, $d$, so that diffusion distances are long. Materials for use at 550°C include

- Some low alloy steels
- Stainless steels
- Titanium alloys
- Iron-based superalloys
- Nickel-based superalloys

E13.5. Pipework with a radius of 20 mm and a wall thickness of 4 mm made of 2 ¼ Cr Mo steel contains a hot fluid under pressure. The pressure is 10 MPa at a temperature of 600°C. The table lists the creep constants for this steel. Calculate the creep rate of the pipe wall, assuming steady-state power-law creep.

<table>
<thead>
<tr>
<th>Material</th>
<th>Reference strain rate $\dot{\varepsilon}_o$ (1/s)</th>
<th>Reference stress $\sigma_o$ (MPa)</th>
<th>Rupture exponent $m$</th>
<th>Activation energy $Q_c$ (kJ/mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 ¼ Cr Mo steel</td>
<td>$3.48 \times 10^{-10}$</td>
<td>169</td>
<td>7.5</td>
<td>280</td>
</tr>
</tbody>
</table>

**Answer.** The creep rate is $1.56 \times 10^{-5}$/sec

E13.6. There is concern that the pipework describe in the previous exercise might rupture in less than the design life of 1 year. If the Monkman Grant constant for 2 ¼ Cr Mo steel is 0.06, how long will it last before it ruptures?

**Answer.** The concern is justified. The rupture time is 4050 hours (169 days).

E13.7. If the creep rate of a component made of 2 ¼ Cr Mo steel must not exceed $10^{-8}$/sec at 500°C, what is the greatest stress that it can safely carry? Use the data listed in the previous two examples to find out.

**Answer.** The stress is 105 MPa
**E13.8.** A stainless steel suspension cable in a furnace is subjected to a stress of 100 MPa at 700°C. Its creep rate is found to be unacceptably high. By what mechanism is creeping? What action would you suggest to tackle the problem? The figure shows the deformation mechanism map for the material.

**Answer.** The temperature and stress on the cable cause it to deform by power law creep. Materials that best resist power law creep are those with high melting points, since diffusion and thus creep-rates scale as $T / T_m$, and a microstructure that maximizes obstruction to dislocation motion through alloying to give a solid solution and precipitate particles.

**E13.9.** The wall of a pipe of the same stainless steel as that of the previous exercise carries a stress of 3 MPa at the very high temperature of 1000°C. In this application it, too, creeps at a rate that is unacceptably high. By what mechanism is creeping? What action would you suggest to tackle the problem?

**Answer.** The temperature and stress on the pipe wall cause it to deform by diffusional flow. Materials that best resist diffusional flow are those with a high melting temperature and a large grain size, $d$, so that diffusion distances are long. Single crystals are best of all; they have no grain boundaries to act as sinks and sources for vacancies, so diffusional creep is suppressed completely.

**E13.10.** It is proposed to make a shelf for a hot-air drying system from Acrylic sheet. The shelf is simply supported, as in the diagram, and has a width $w = 500$ mm, a thickness $t = 8$ mm and a depth $b = 200$ mm. It must carry a distributed load of 50 N at 60°C with a design life of 8000 hours (about a year) of continuous use. Use the creep modulus plotted in Figure 13.17 and the solution to the appropriate elastic problem (Chapter 5) to find out how much it will sag in that time.

**Answer.** The deflection is $\delta = \frac{F w^3}{C_I E_c I}$ where $C_I = 384/5$, $I = \frac{bt^3}{12}$ and $E_c$ is the creep modulus. From Figure 13.7 the creep modulus at 60°C and a time of 8000 hours ($3 \times 10^7$ seconds) is just over 0.1 GPa. Inserting the values of $w$, $b$ and $t$ gives a deflection of 8 mm.
Exploring design with CES EduPack (use Level 2 unless otherwise stated).

**E13.11.** Use the Search facility in CES EduPack to find

Materials for turbine blades

Materials for thermal barrier coatings

**Answer.**

(a) Turbine blades

- Nickel alloys
- Titanium alloys
- Nickel-based superalloys
- Zirconia
- Silicon nitride

(b) Thermal barrier coatings

- Silica glass
- Alumina
- Zirconia

**E13.12.** Use the CES EduPack software to make a bar chart for the glass temperatures of polymers like that for melting point in the text. What is the range of glass temperatures for polymers?

**Answer.** The range is approximately –100 C to +200 C.

**E13.13.** The analysis of thermal barrier coatings formulated the design constraints. Use the selector to find the best material. Here is a summary of the constraints and the objective.

<table>
<thead>
<tr>
<th>Function</th>
<th>Thermal barrier coating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraints</td>
<td>Maximum service temperature &gt; 1300 C</td>
</tr>
<tr>
<td></td>
<td>Adequate strength: $\sigma_y &gt; 400$ MPa</td>
</tr>
<tr>
<td>Objective</td>
<td>Minimize thermal conductivity $\lambda$</td>
</tr>
<tr>
<td>Free variable</td>
<td>Choice of material</td>
</tr>
</tbody>
</table>

**Answer.** Five materials – four ceramics and one metal – meet the constraints on service temperature and strength:

- Alumina
- Boron carbide
- Silicon carbide
Zirconia
Tungsten alloys

A bar chart of Thermal conductivity then show the Zirconia ZrO₂ has by far the lowest thermal conductivity. It is, in fact, the principal material used for thermal barrier coatings.

E13.14. Find materials with a maximum service temperature $T_{\text{max}} > 500$ C and the lowest possible thermal conductivity. Switch the database to Level 3 to get more detail. Report the three materials with the lowest thermal conductivity.

**Answer.**

Zirconia (Transformation toughened)(L)
Zirconia (Y-TZP)(HIP)
Zirconia (yttria stabilized, transformation toughened)

**Exploring the science with CES EduPack Elements**

**E13.15.** The claim is made in the text (equation 13.12b) that the activation energy for diffusion, normalized by $RT_m$, is approximately constant for metals:

$$\frac{Q_d}{RT_m} \approx 18$$

Make a bar chart of this quantity and explore the degree to which it is true.

**Answer.** The bar chart supports the claim. The normalized activation energy spans the range

$$10 < \frac{Q_d}{RT_m} < 60$$

but the majority lie between 16 and 20.
E13.16. The claim is made in the text (equation 13.12c) that the diffusion coefficient of metals, evaluated at their melting point, is also approximately constant:

\[ D_{T_m} = D_o \exp\left(\frac{Q_d}{RT_m}\right) \approx 10^{-12} \text{ m}^2 / \text{s} \]

Make a bar chart of this quantity and explore the degree to which it is true. If a material is held just below its melting point for 1 hour, what, approximately, is the diffusion distance?

Answer. The bar-chart shows that the value \( D_{T_m} \approx 10^{-12} \text{ m} \) lies in the middle of the range of range of \( D_{T_m} \) for the elements.

E13.17. What is a “typical” value for the pre-exponential, \( D_o \)? Is it roughly constant for the elements? Make a chart with the activation energy for diffusion, \( Q_d \), on the x-axis and the pre-exponential \( D_o \) on the y-axis to explore this.

Answer. The great majority of the elements have pre-exponentials that lie close to \( D_0 = 10^{-4} \text{ m}^2 / \text{s} \).
**E13.18.** The diffusive jump shown in Figure 13.9 requires that the diffusing atom breaks its bonds in its starting position in order to jump into its final one. You might, then, expect that there would be at least an approximate proportionality between the activation energy for diffusion $Q_d$ and the cohesive energy $H_c$ of the material. Make a chart with $H_c$ on the x-axis and $Q_d$ on the y-axis. Report what you find.

**Answer.** The plot shows $Q_d$ and $H_c$, both in units of kJ/mol. The line is a fit to the data. Its equation is

$$Q_d = \frac{H_c}{2}$$

This is an interesting result: the activation energy for diffusion is very large, equal to half the energy needed to separate the atoms completely.
This is from Granta's Teaching Resources website, which aims to support teaching of materials-related courses in Engineering, Science and Design. The resources come in various formats and are aimed at different levels of student. As well as the solution manuals to Professor Mike Ashby's textbooks the following resources are also available:

**Book 1:** Getting Started with CES EduPack

**Book 2:** Material and Process Selection Charts

**Book 3:** Useful Approximate Solutions for Standard Problems

**Book 4:** PowerPoint Lectures

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